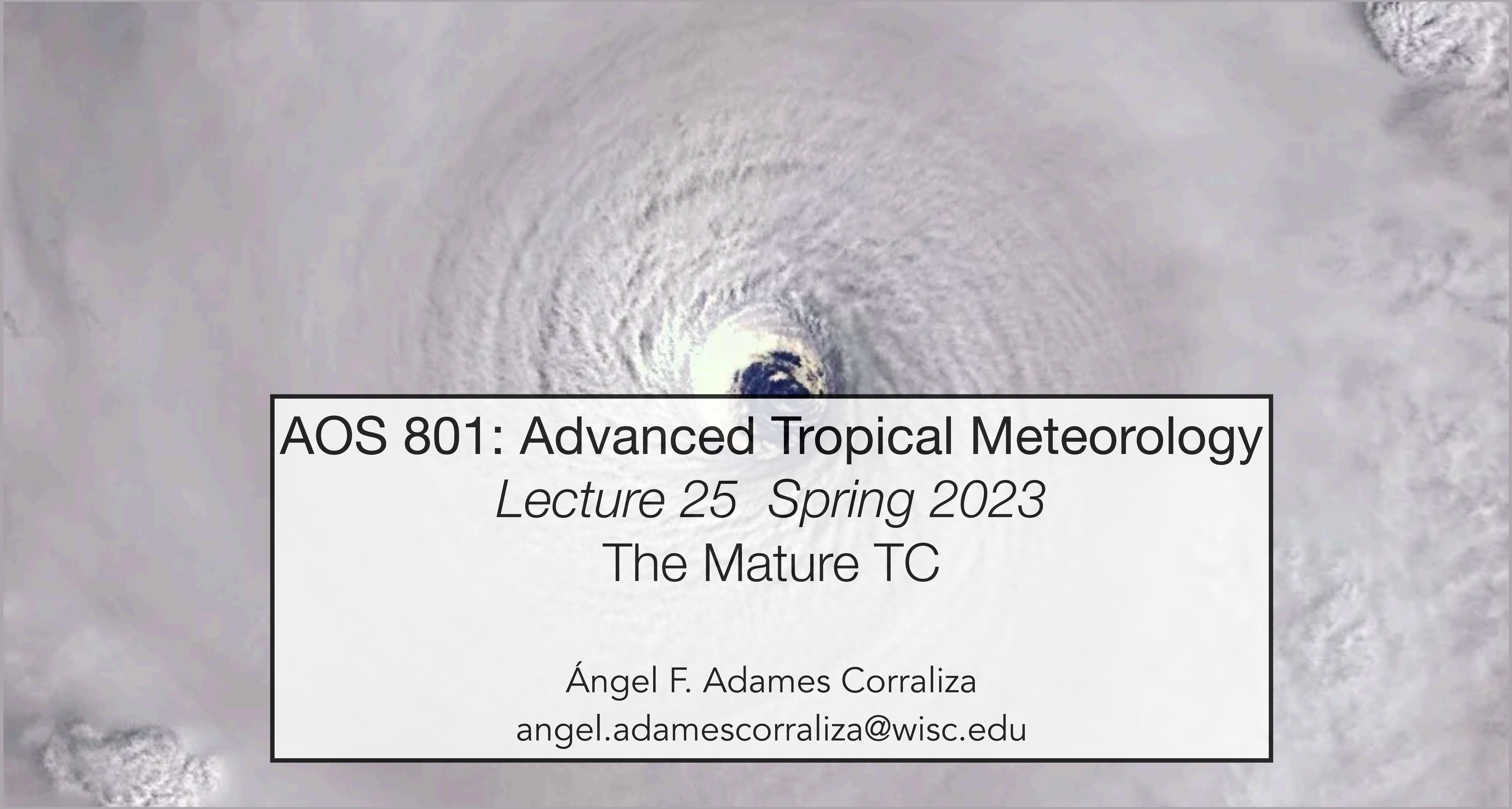


AOS 801: Advanced Tropical Meteorology

Lecture 25 Spring 2023

The Mature TC

Ángel F. Adames Corraliza
angel.adamescorraliza@wisc.edu

Announcements

HW5 and PA5 are due next Tuesday.

Guidelines to your final presentation are now on Canvas.

Final lectures are this week.

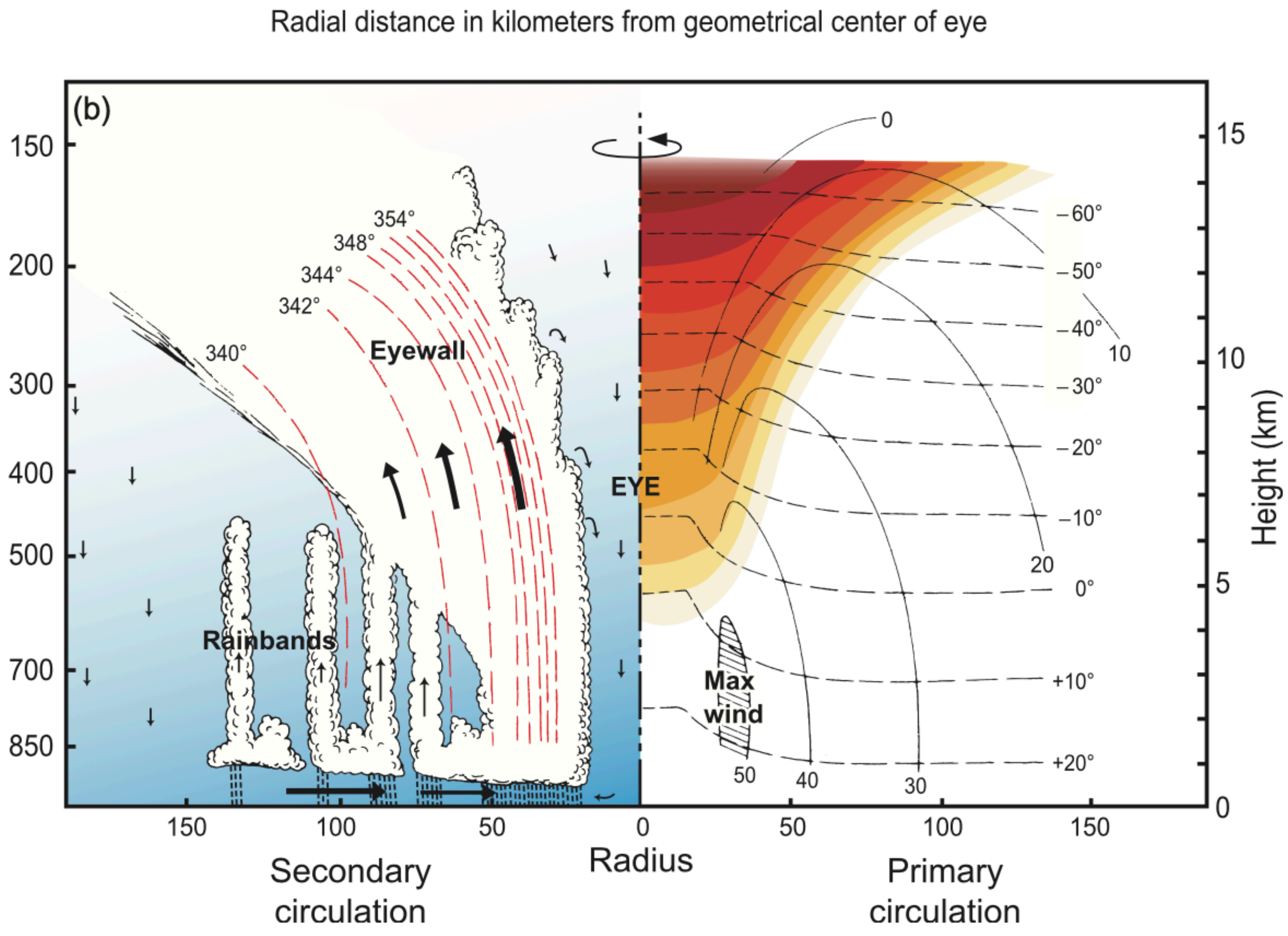
Gradient Balance

It is convenient to write the TC's tangential circulation in terms of "absolute angular momentum"

$$M = vr + \frac{1}{2} f r^2$$

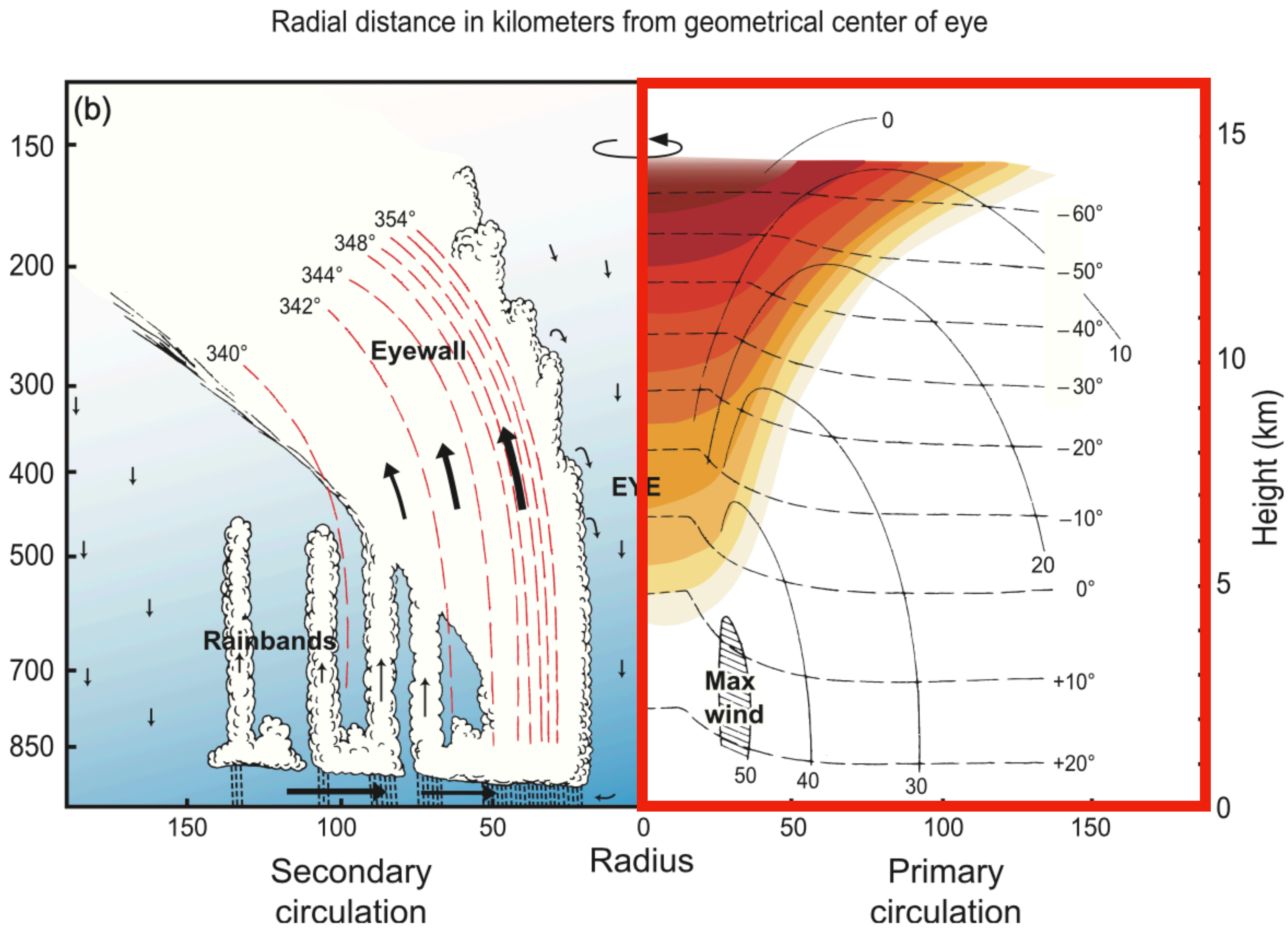
So that the thermal wind equation becomes:

$$\frac{\partial M^2}{\partial p} = -\frac{R_d r^3}{p}\frac{\partial T}{\partial r}$$



Houze (2014)

Gradient Balance



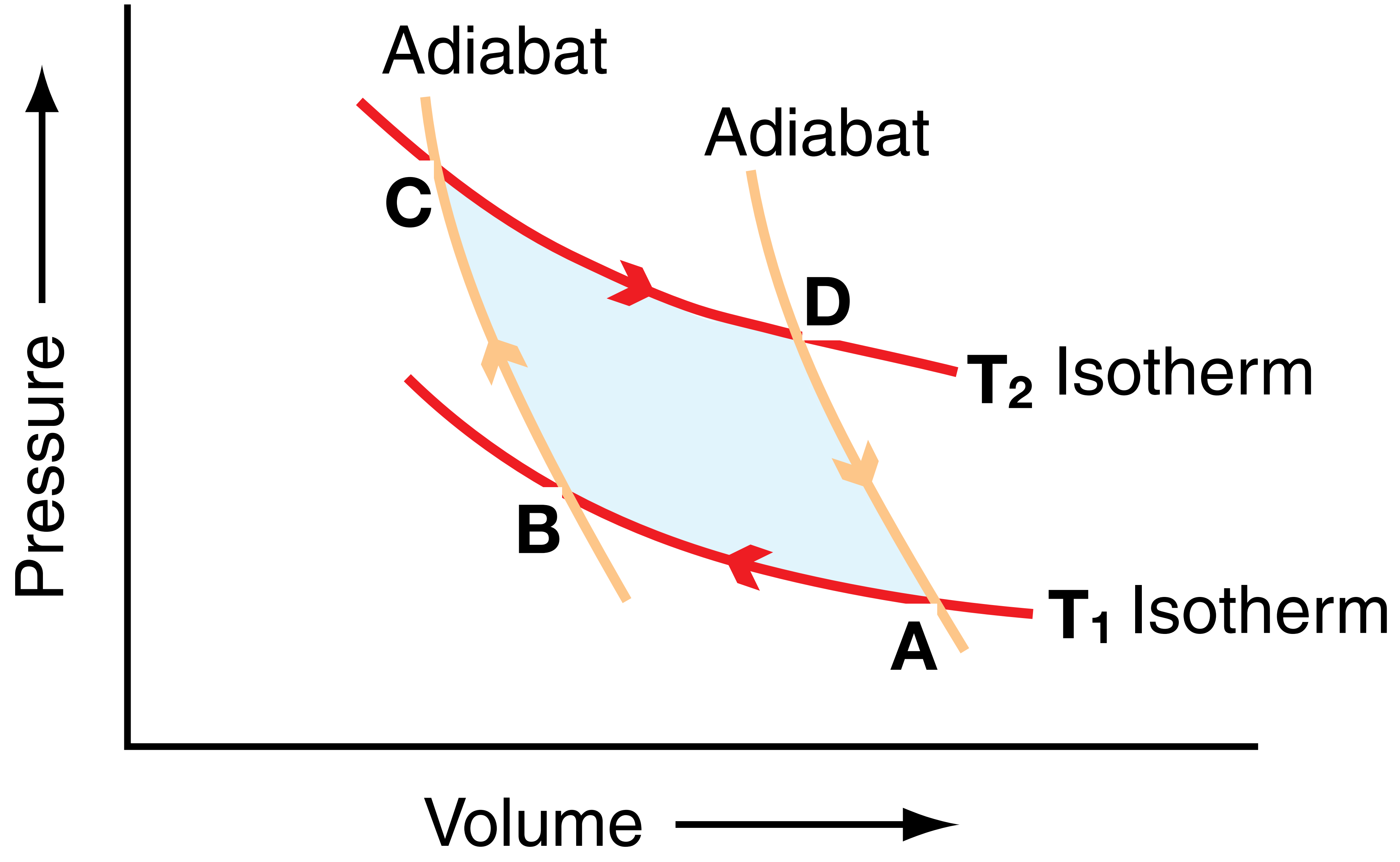
So far we have focused on this side of the picture

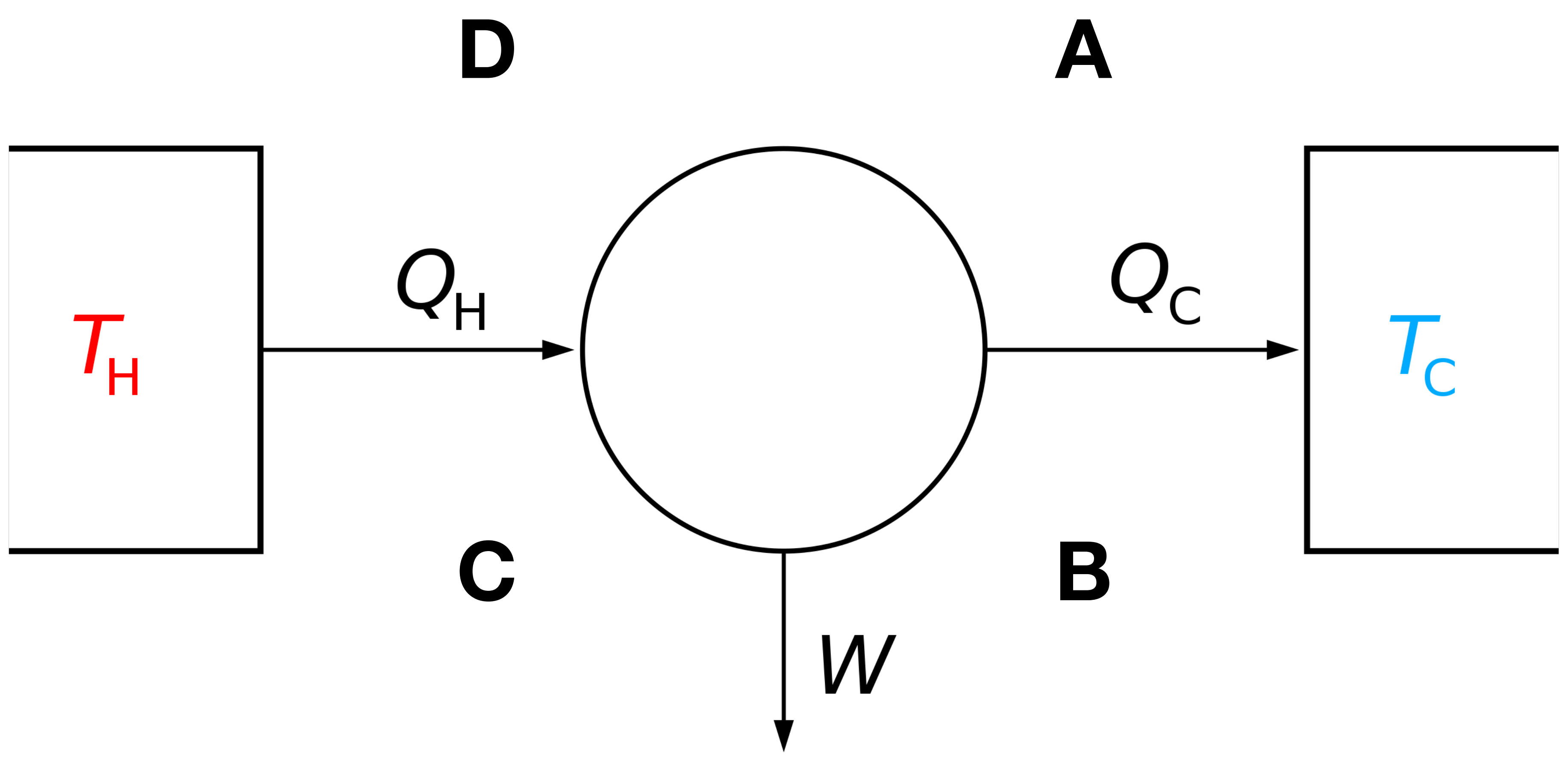
Houze (2014)

Consider a cyclical process involving an ideal gas.

The cycle goes through four steps, as outlined in the diagram on the right.

Essentially, you input heat at a high temperature and the system does work in proportion to the amount of input heat.





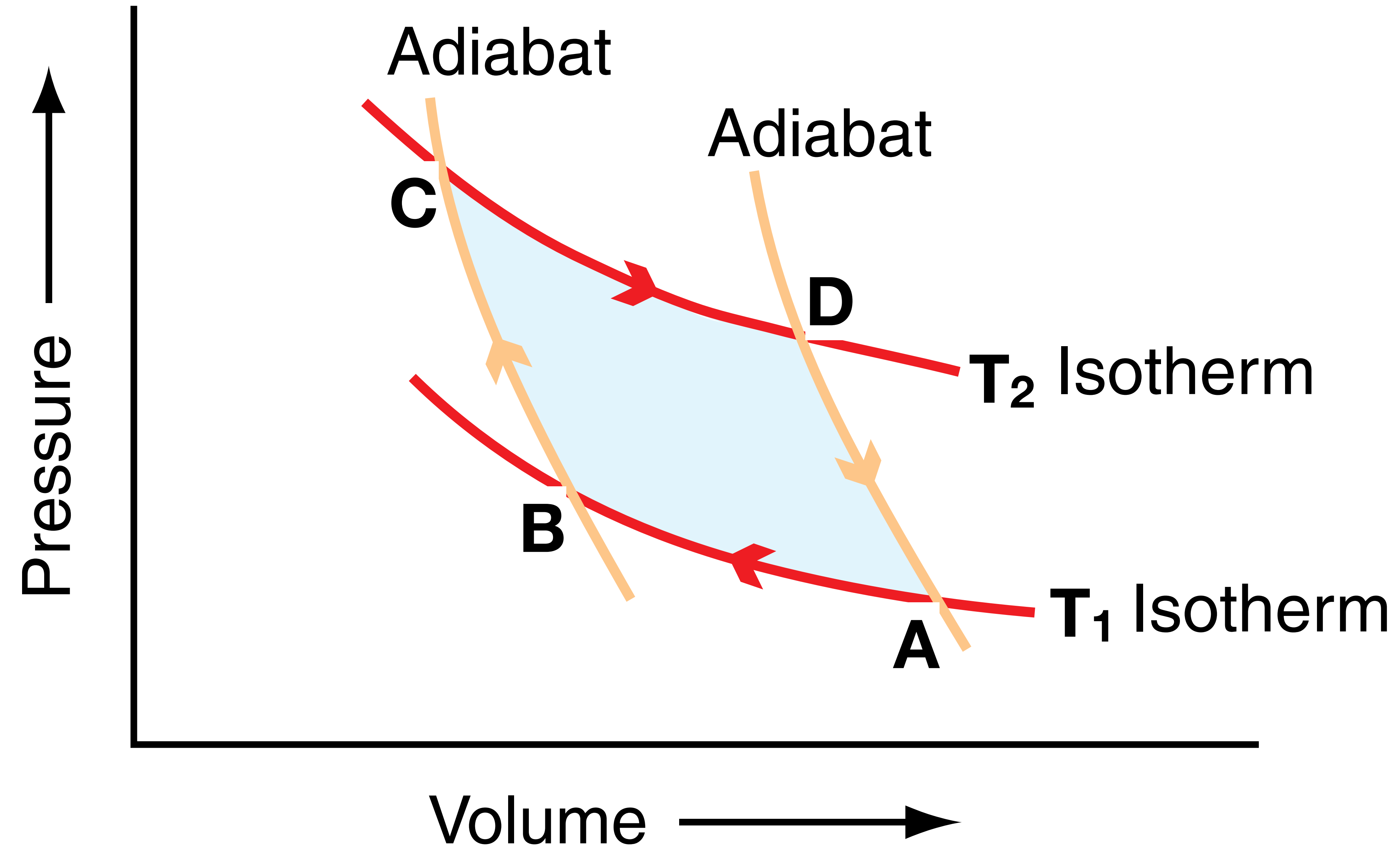
Carnot Engine

The first law (internal energy form) integrated over this cycle takes the form:

$$\oint C_v dT = \oint \delta Q - \oint \delta W$$

Because state variable don't change during a closed loop integral, it follows that

$$\oint \delta Q = \oint \delta W$$

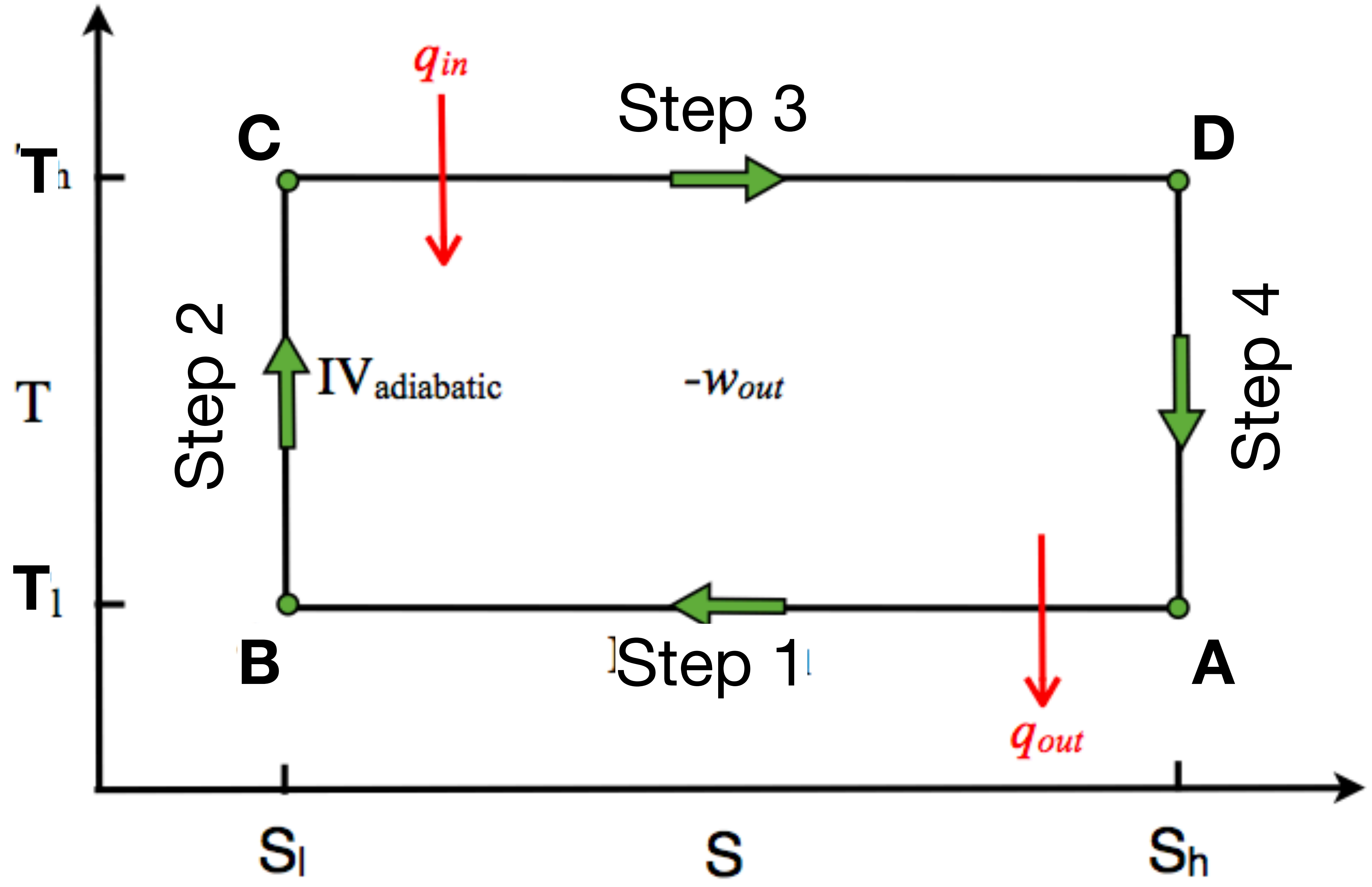


Writing these in exact derivative form we have

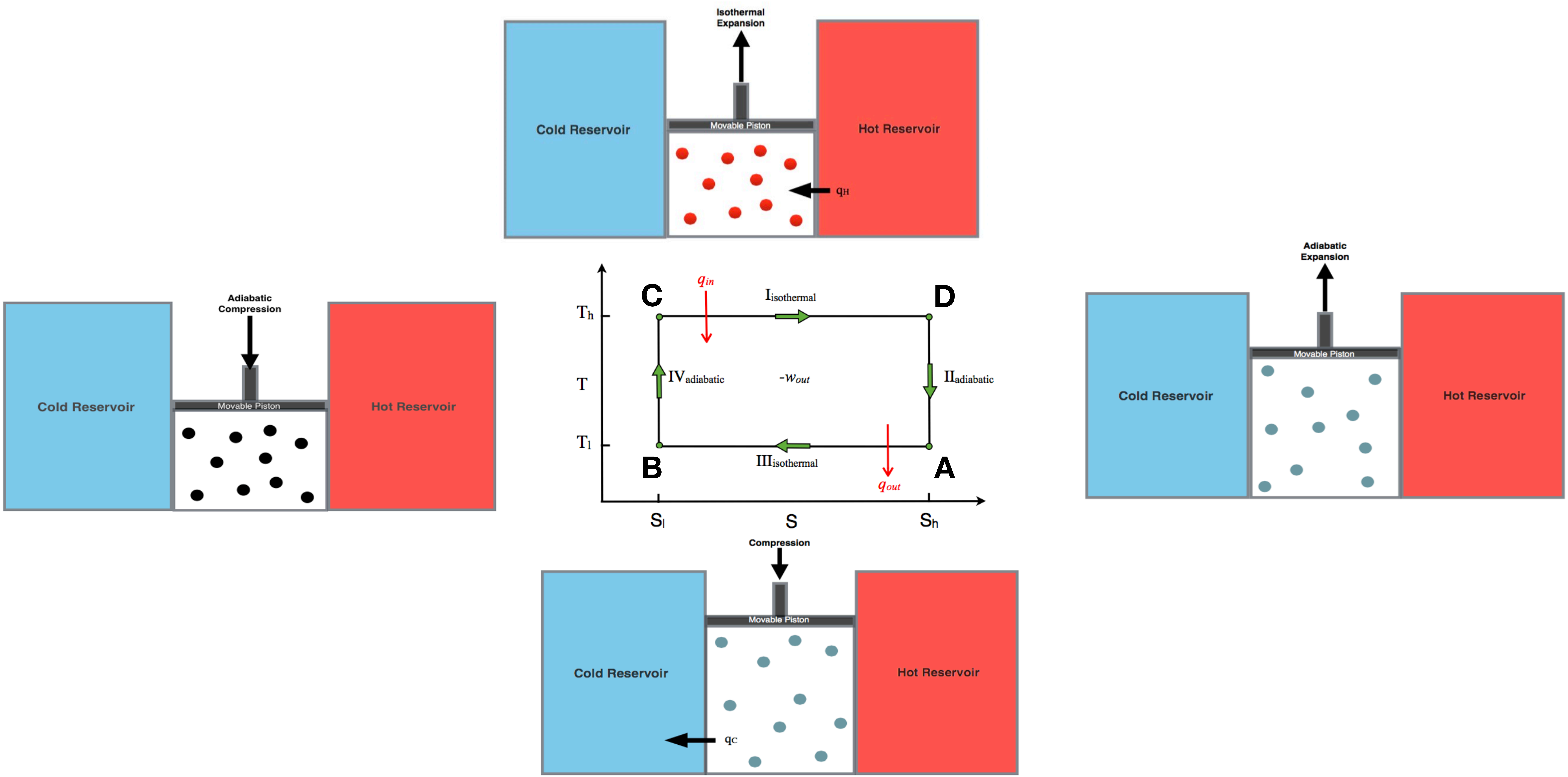
$$\oint T ds = \oint p d\alpha \neq 0$$

Now let's consider a cycle divided into 4 steps:

1. Isothermal compression at a cooler T1
2. Adiabatic compression to T2
3. Isothermal expansion at T2
4. Adiabatic expansion back to T1



Carnot Engine

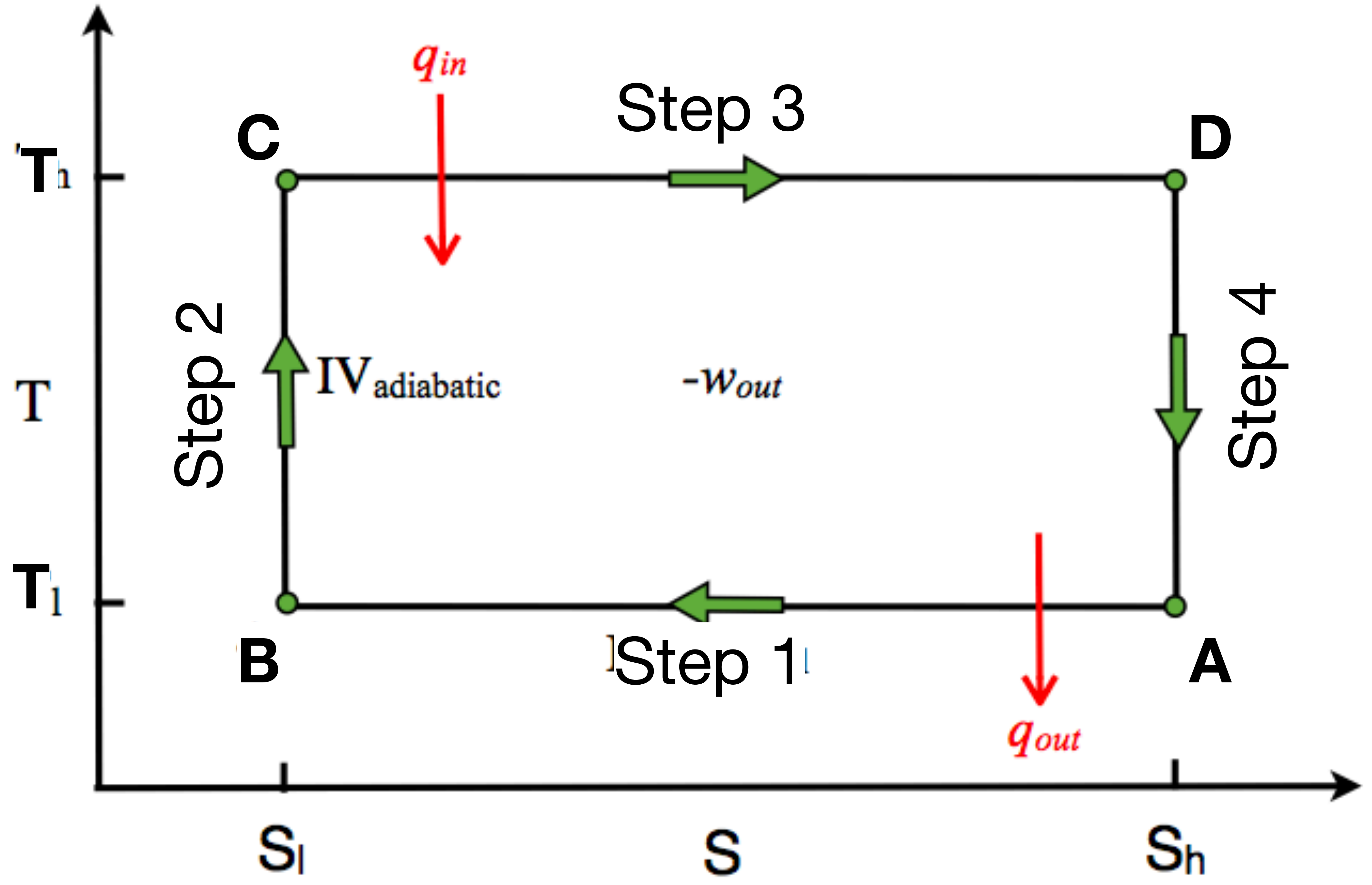


By expanding the integral into the four components of the cycle we find that

$$W = \oint p d\alpha = q_{in} - q_{out} = \varepsilon T_1 (s_{in} - s_{out})$$

$$\varepsilon = \frac{\textbf{Work done}}{\textbf{Heat absorbed}} = \frac{T_2 - T_1}{T_2}$$

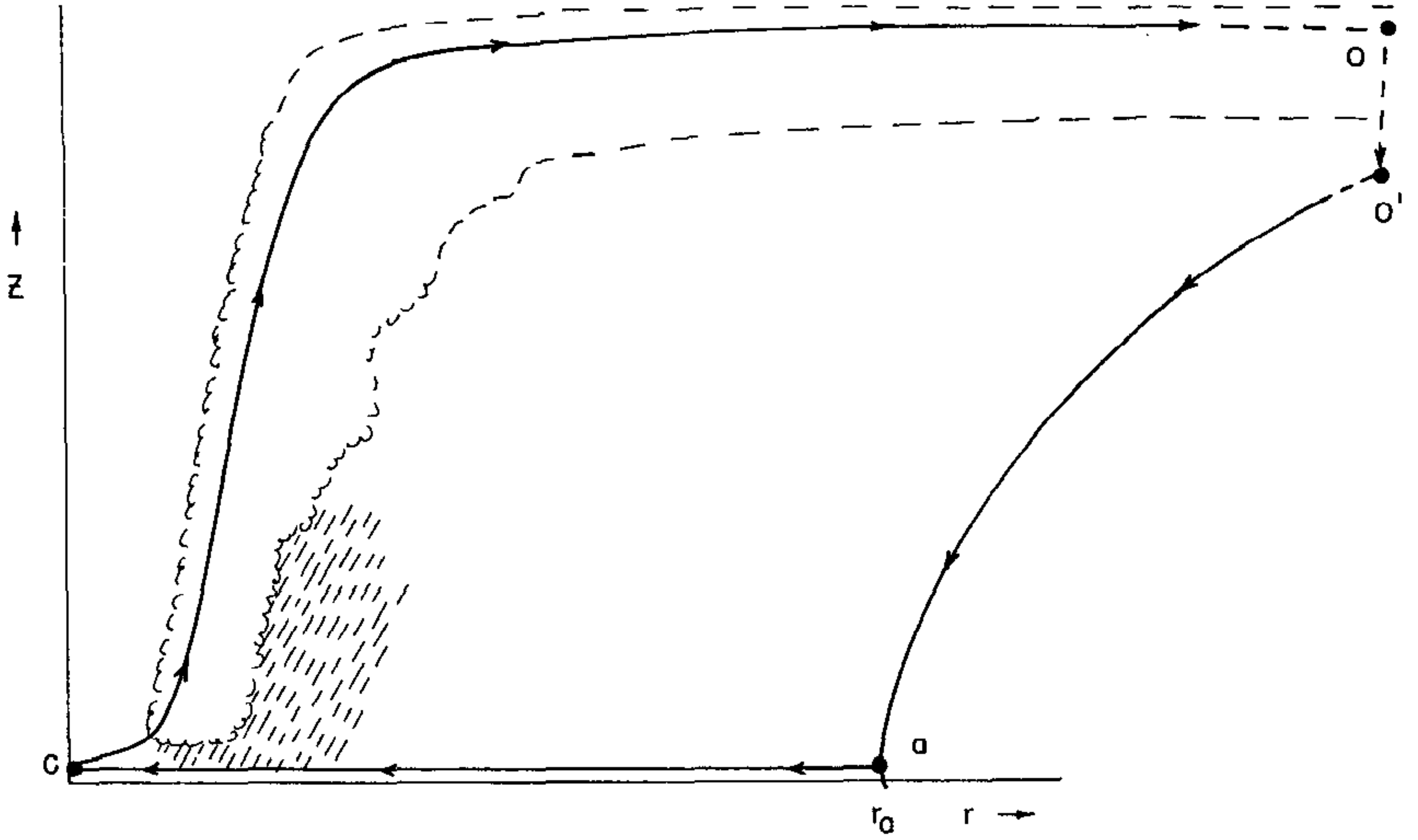
Is the Carnot Efficiency



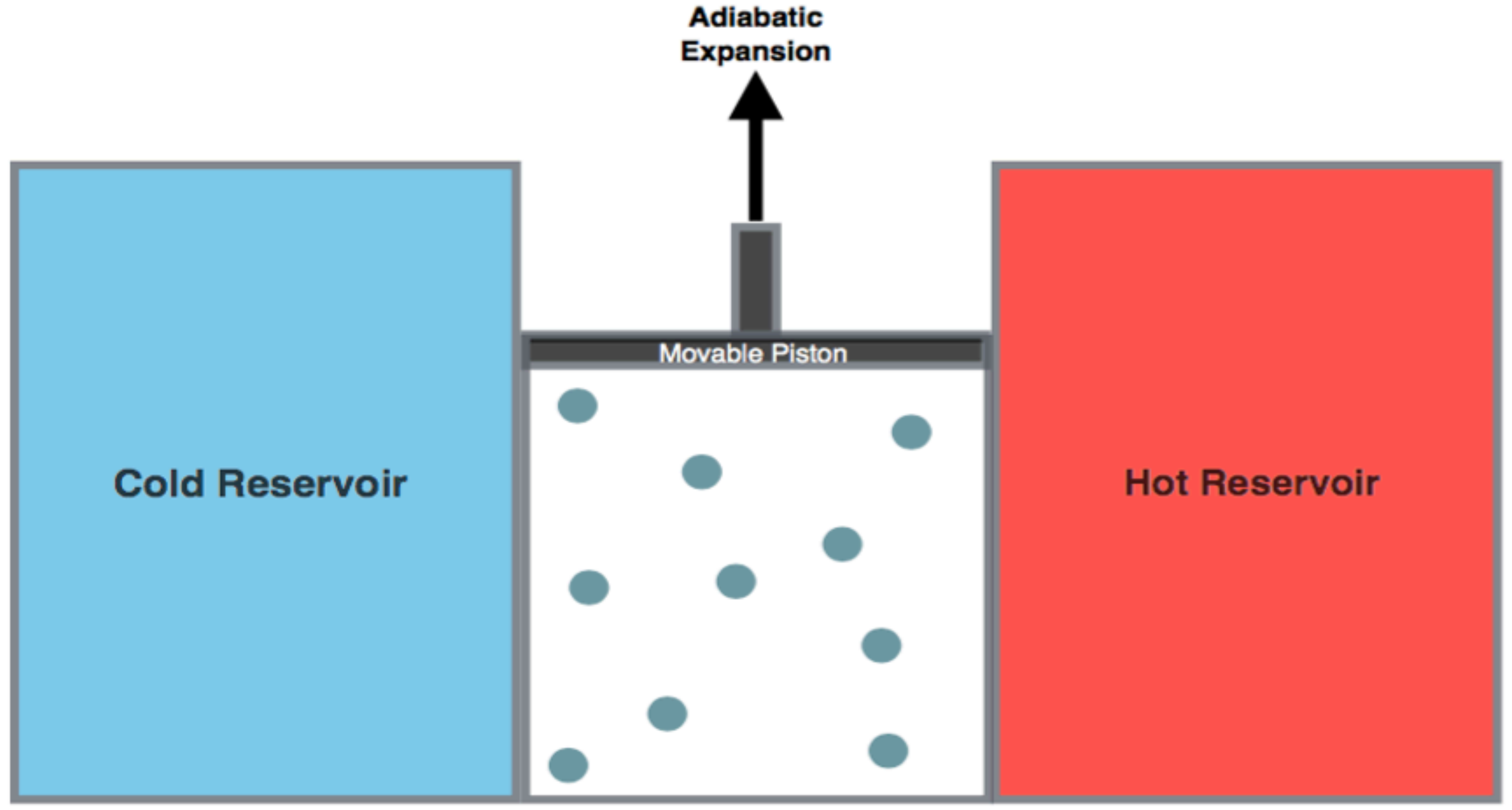
The mature TC as a Carnot Engine

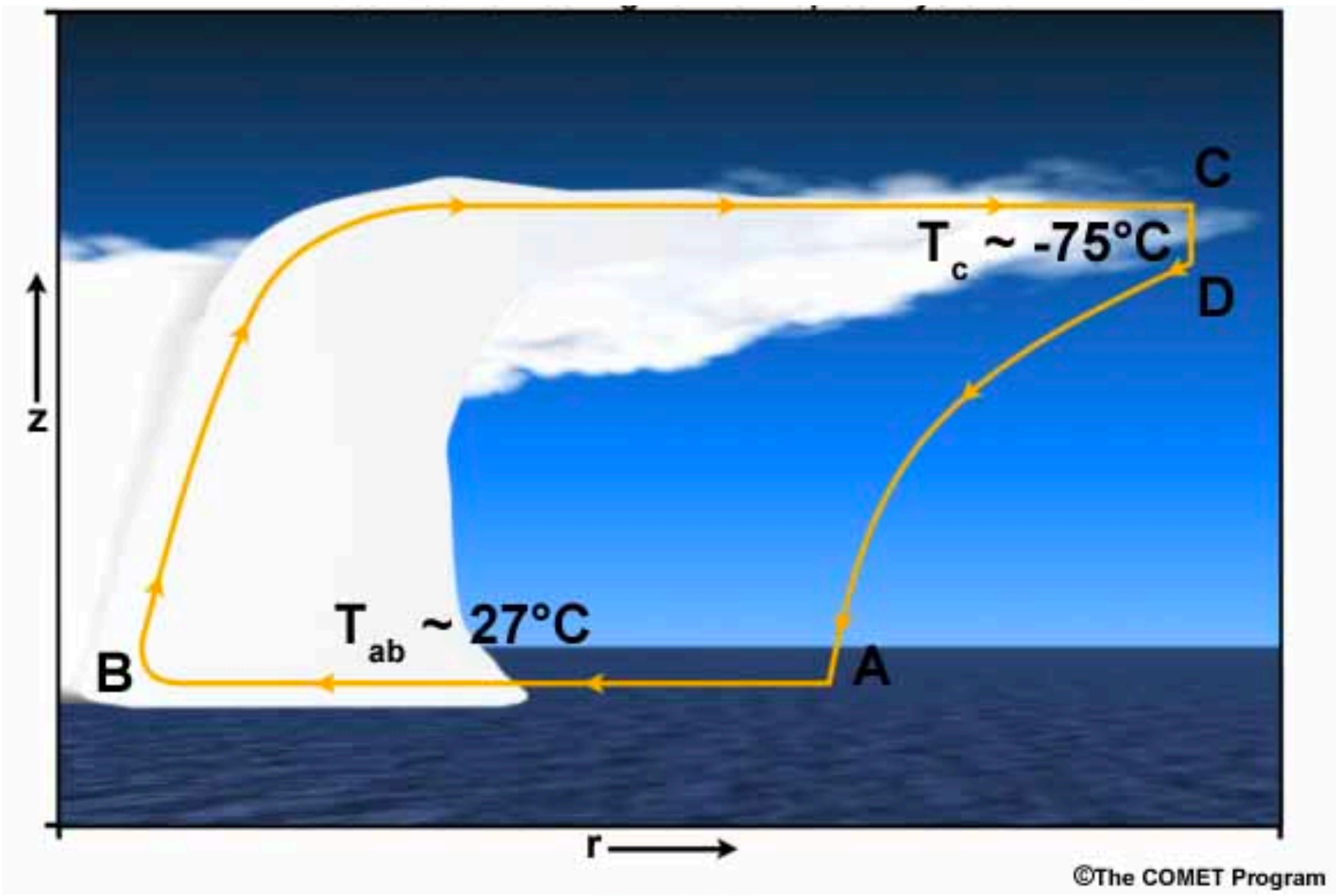
We can interpret the TC as a Carnot engine but using moist entropy instead of dry entropy.

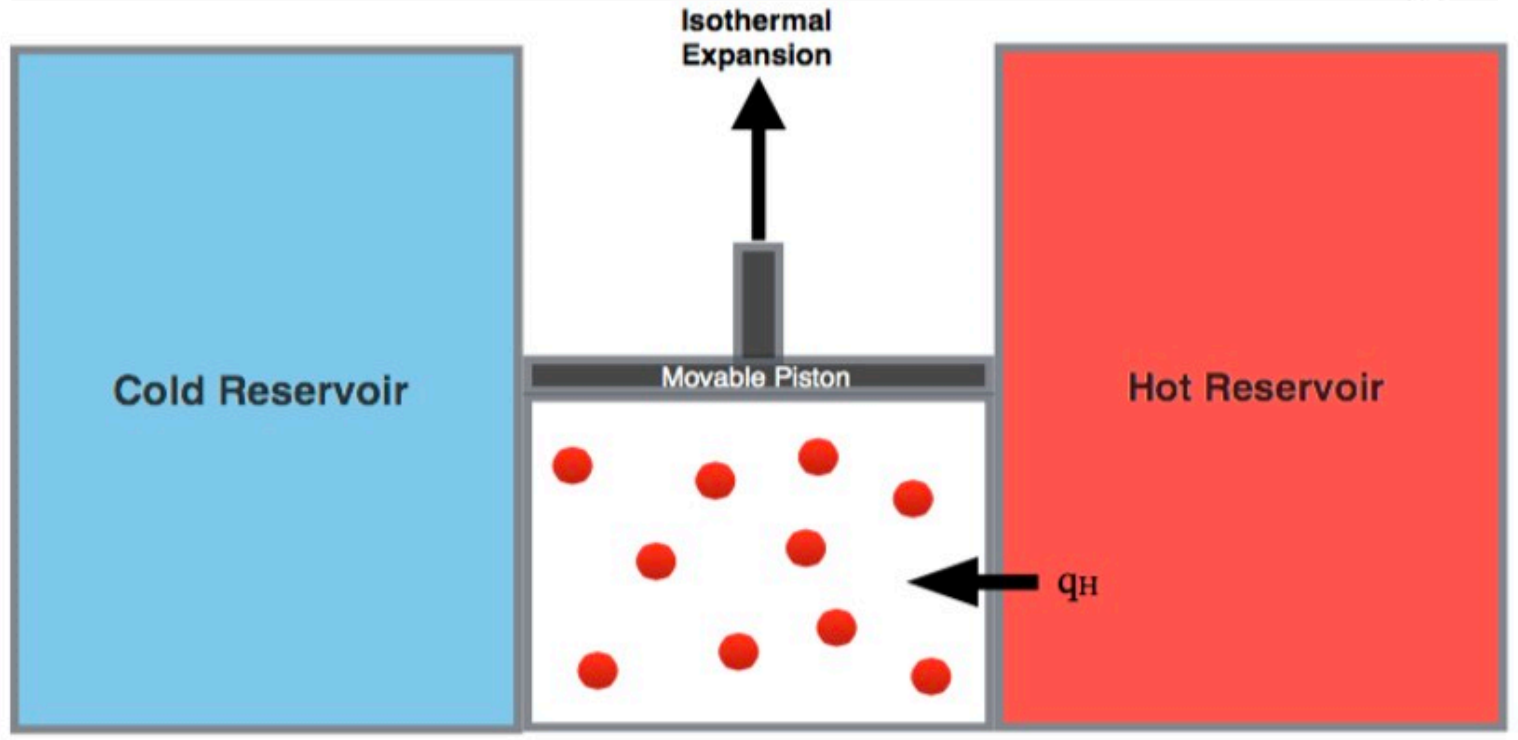
$$s_m = C_p \ln \theta_e + \text{const}$$

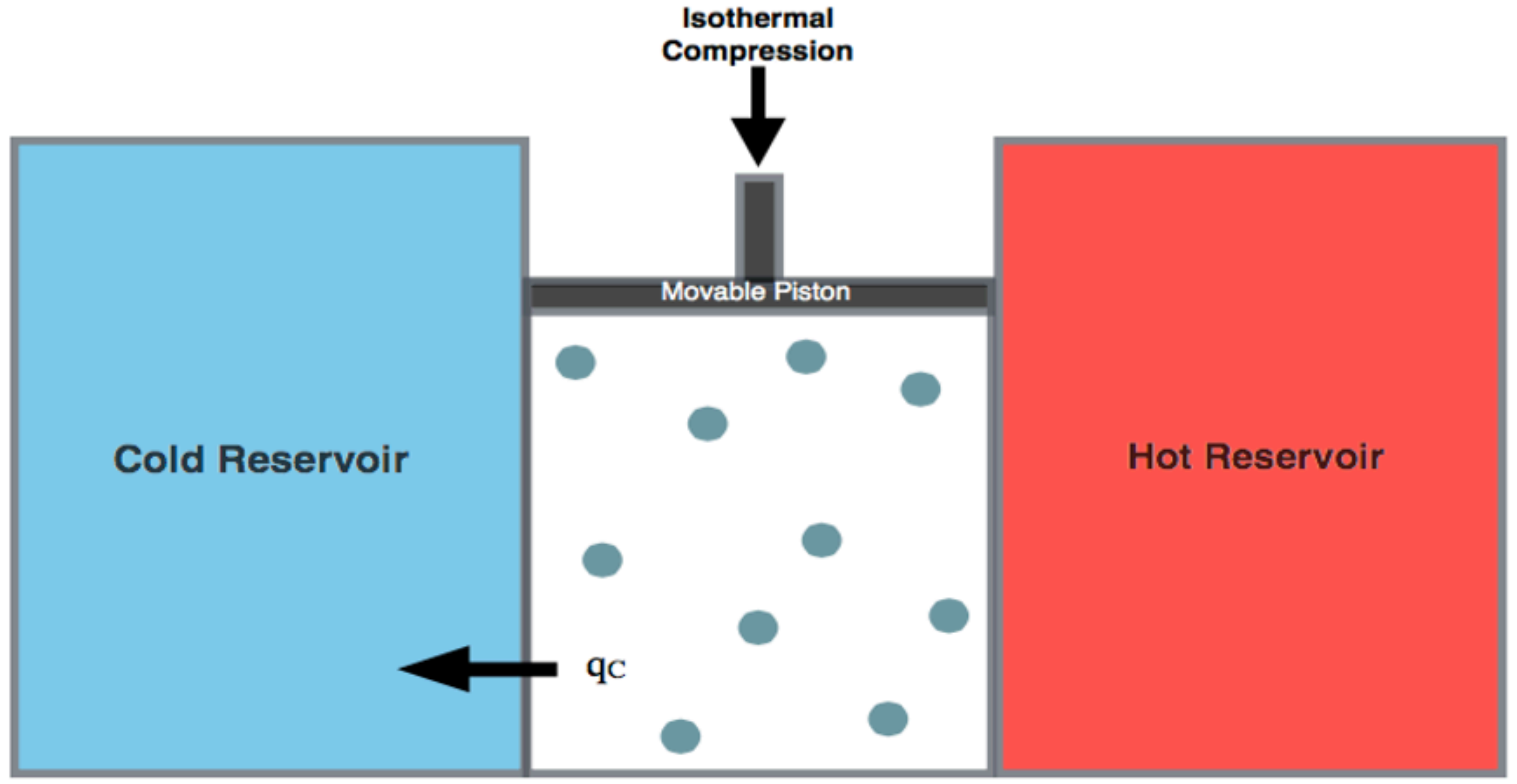


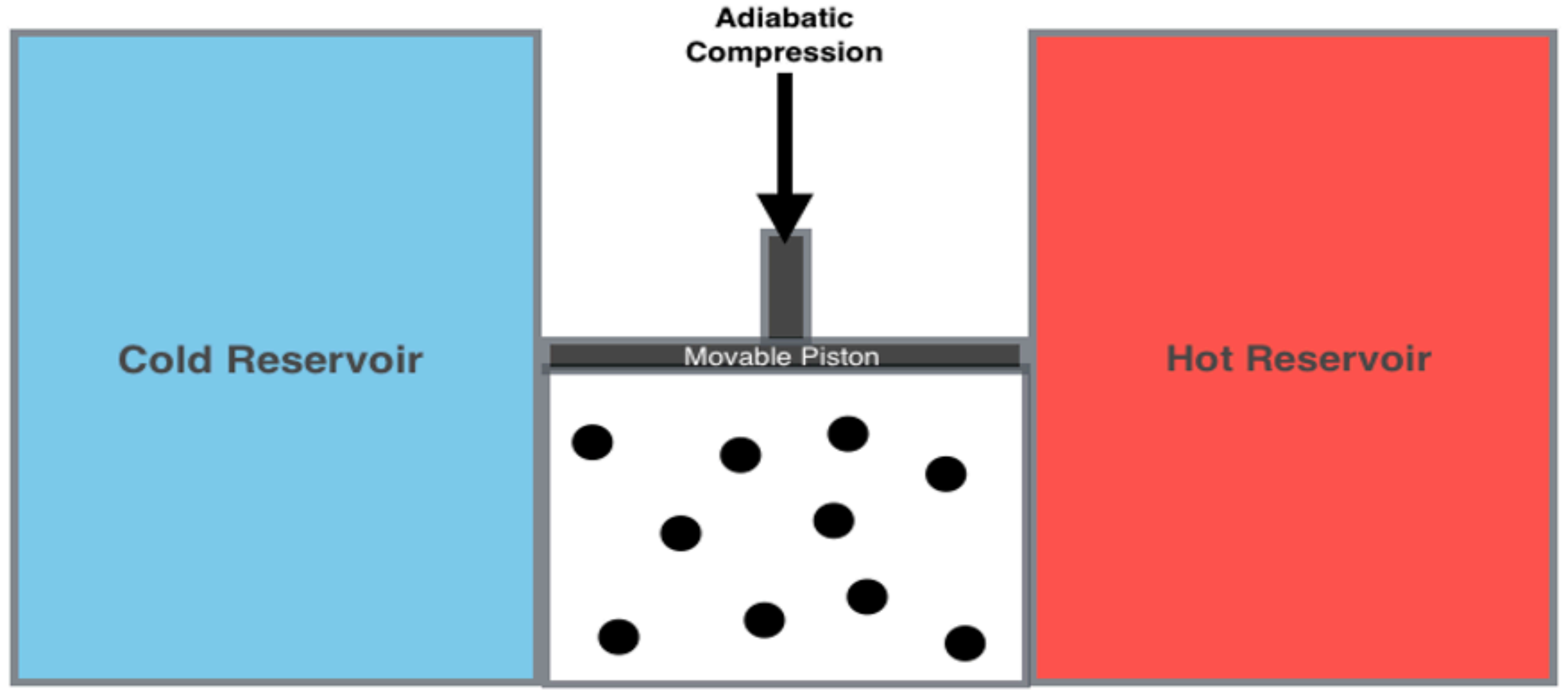
The mature TC as a Carnot Engine



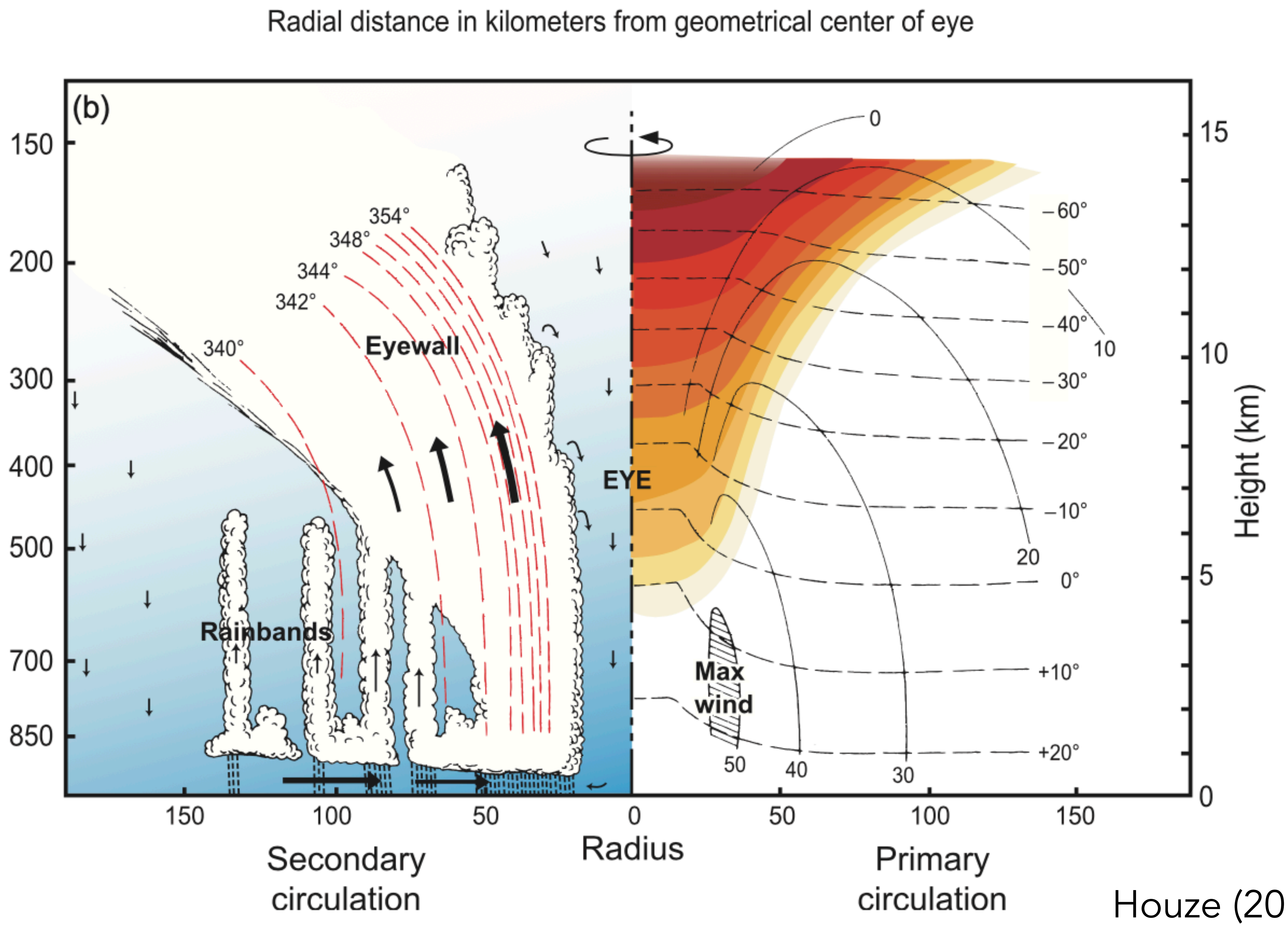








Relation between primary and secondary circulation



Houze (2014)

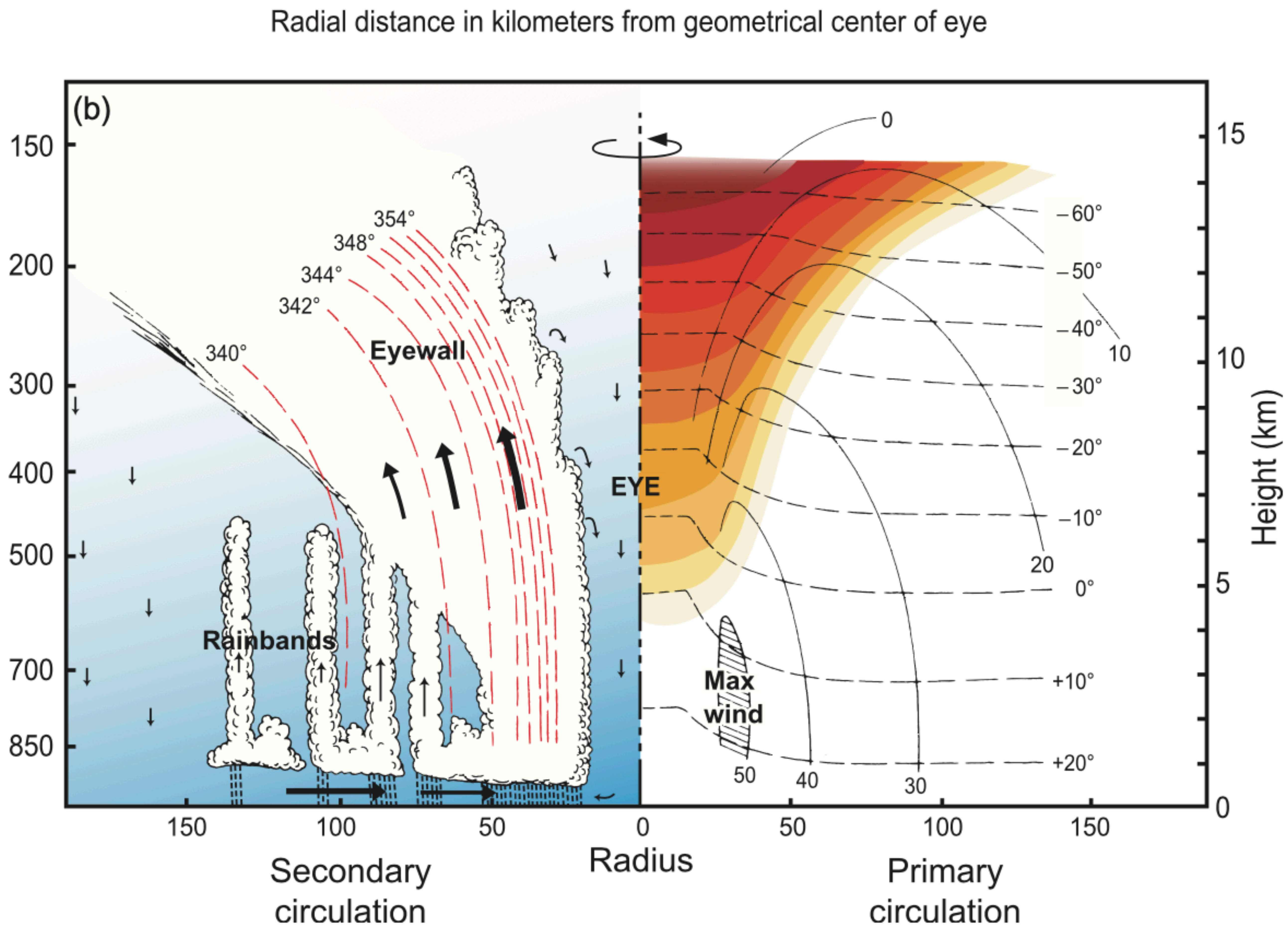
Thermal Wind

It is convenient to write the TC's tangential circulation in terms of "absolute angular momentum"

$$M = vr + \frac{1}{2} f r^2$$

So that the thermal wind equation becomes:

$$\frac{\partial M^2}{\partial p} = -\frac{R_d r^3}{p}\frac{\partial T}{\partial r} = -r^3 \frac{\partial \alpha}{\partial r}$$



Houze (2014)

Thermal Wind in the TC eyewall

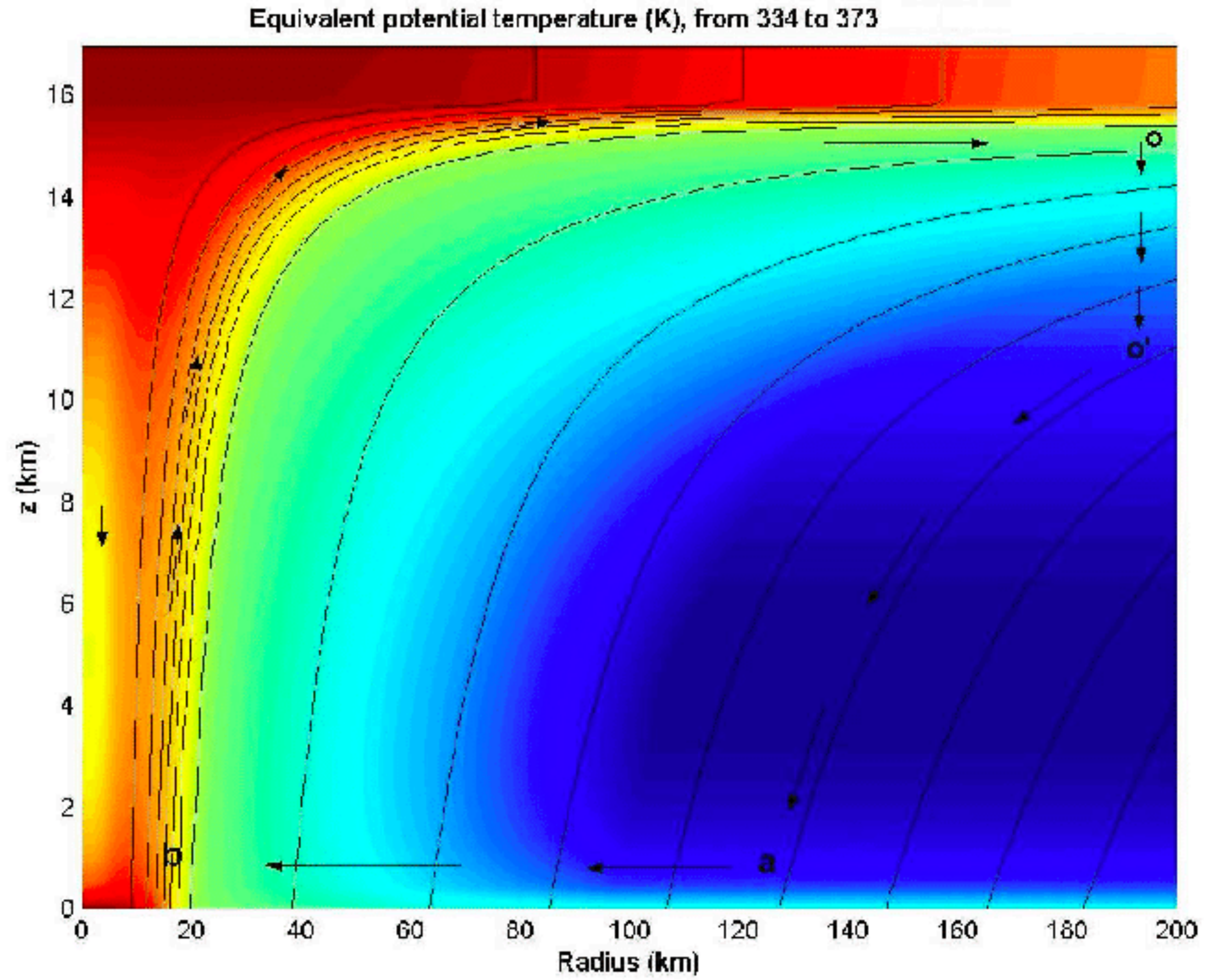
Using Maxwell's relation we write the thermal wind as:

$$\frac{\partial M^2}{\partial p} = -\,r^3\left(\frac{\partial T}{\partial p}\right)_{s_m}\left(\frac{\partial s_m}{\partial r}\right)_p$$

Let's assume that angular momentum surfaces are congruent with moist entropy surfaces, i.e. $M(s_m)$. The radius of the eyewall is assumed to be a function of pressure:

$$\frac{2M}{r^3}\left(\frac{\partial r}{\partial p}\right)_M = \left(\frac{\partial T}{\partial p}\right)_M\left(\frac{\partial s_m}{\partial M}\right)_M$$

Note: the sloping angular momentum and moist entropy surfaces are the reason we see the stadium effect.



Maximum potential intensity

Assuming that they TC eyewall is in cyclostrophic balance, then we can solve the equation for the radius to obtain the "maximum potential intensity"

$$v_{max}^2 = \frac{C_h}{C_d}\frac{T_s - T_o}{T_o}\left(h_s^* - h_a\right)$$

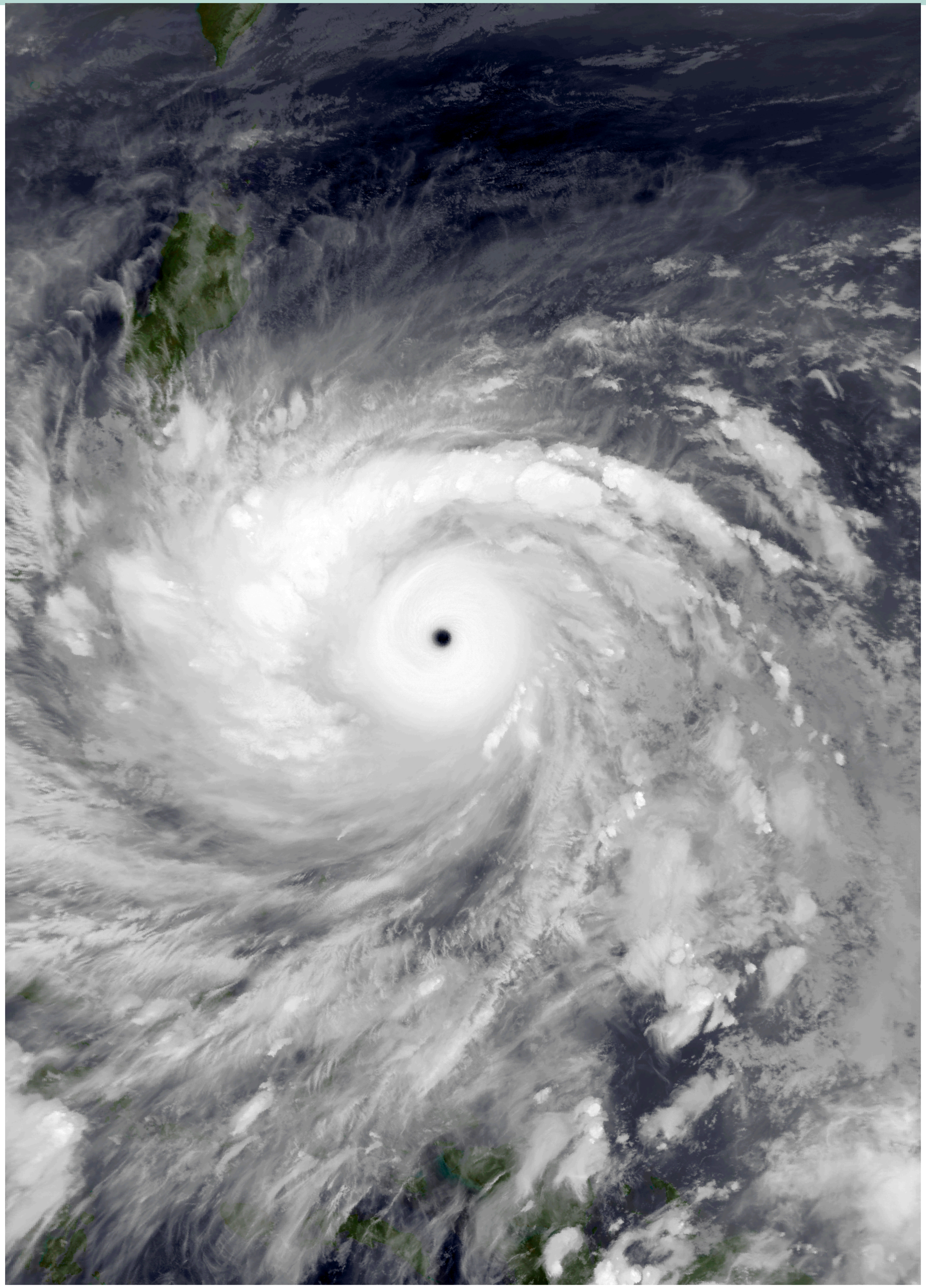
Maximum potential intensity (MPI)

Assuming that the temperature difference between the air and the ocean is small, the formula can be simplified to

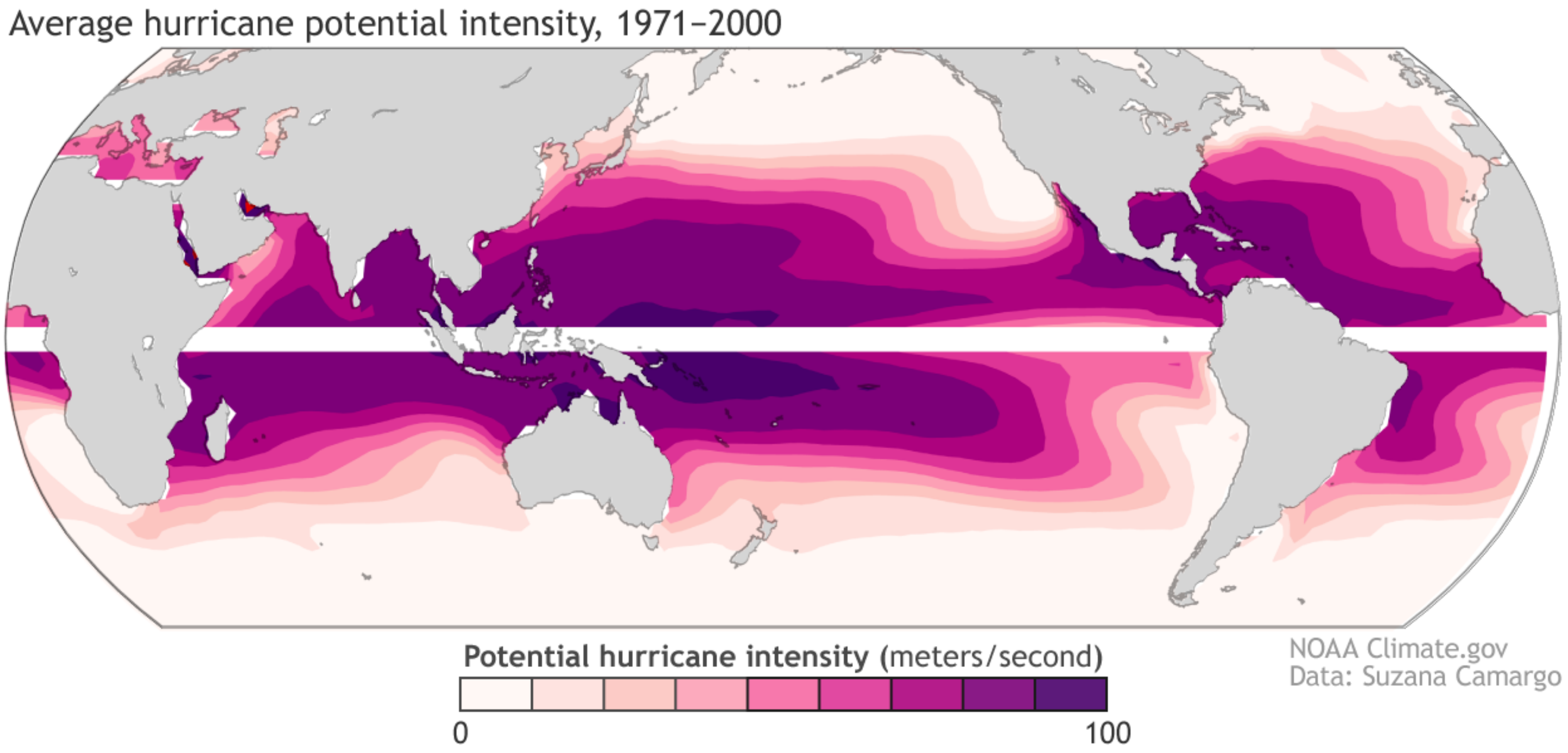
$$v_{max}^2 = \frac{C_h}{C_d}\frac{T_s - T_o}{T_o} L_v q_s^* \,(1 - \mathrm{RH})$$

If the boundary-layer RH is similar everywhere (observations support this), then the temperature of the sea surface predominantly determines the maximum intensity of a TC.

What would happen to the MPI if the TC suddenly upwelled cold water?



MPI climatology



It's been fun lecturing you