First law of thermodynamics is

$$C_v \frac{DT}{Dt} = Q - W$$

diabatic heating

work done by system

The idea of the Carnot cycle is that you do a loop integral in the cycle. For this it's convenient to write the first law in infinitesimal form:

$$C_v\, dT = \delta Q - \delta W$$

state variable

inexact derivative

corresponds to infinitesimals of process variables

The Carnot cycle implies that:

$$\oint \cancel{C_v dT} = \oint \delta Q - \oint \delta W$$

no change during cycle, so the integral becomes

$$\oint \delta Q = \oint \delta W$$

$$Q = W$$

diabatic heating = work done by system

Recall that

$$Q_{Te} = T_e\, dS_e = T_e (S_e - S_h)$$
$$Q_{Th} = T_h\, dS = T_h (S_h - S_e)$$
$$Q_{Te} + Q_{Th} = T_h (S_h - S_e) - T_e (S_h - S_e)$$
$$= (T_h - T_e)(S_h - S_e)$$

So that $Q_{net} > 0$ and $W_{net} > 0$

The TC max. intensity

Thermal wind : $\frac{\partial M^2}{\partial p} = -r^3 \frac{\partial \alpha}{\partial r}$ $\qquad \alpha = \frac{R_d T}{p}$

spec. volume

Maxwell's relation says:

$$\left.\frac{\partial \alpha}{\partial r}\right|_p = \left(\frac{\partial T}{\partial p}\right)_{S_m} \left.\frac{\partial S_m}{\partial r}\right|_p \qquad S_m = c_p \ln \theta_e + \text{const}$$

So that:

$$\frac{1}{r^3}\frac{\partial M^2}{\partial p} = -\left(\frac{\partial T}{\partial p}\right)_{S_m} \frac{\partial S_m}{\partial r} \qquad \frac{\partial M^2}{\partial p} = 2M\frac{\partial M}{\partial p}$$

Assuming that $S_m = S_m(M)$ and $p \sim p(r)$

we can use the chain rule

$$-\frac{2M}{r^3}\cancel{\frac{\partial M}{\partial r}}\left(\frac{\partial r}{\partial p}\right)_M = -\left(\frac{\partial T}{\partial p}\right)_{S_m}\left(\frac{\partial S_m}{\partial M}\right)_M \cancel{\frac{\partial M}{\partial r}}$$

Note : $\frac{\partial M}{\partial p} = -\frac{\partial M}{\partial r}\frac{\partial r}{\partial p}$ because of a change in direction of the unit vector

you are changing your coordinate system from isobaric to angular momentum based.

$$\frac{2M}{r^3}\left(\frac{\partial r}{\partial p}\right)_M = \left(\frac{\partial T}{\partial p}\right)_M \left(\frac{\partial S_m}{\partial M}\right)_M$$

Note that : $\frac{1}{r^3}\frac{\partial r}{\partial p} = -\frac{1}{2}\frac{\partial}{\partial p}\left(\frac{1}{r^2}\right) = -\frac{1}{2}\frac{\partial r^{-2}}{\partial p}$

$$-M\frac{\partial r^{-2}}{\partial p} = \left(\frac{\partial T}{\partial p}\right)_M \left(\frac{\partial S_m}{\partial M}\right)_M$$

We are now going to integrate over the TC column

$$M \int_{P_o}^{P_s} \frac{\partial r^{-2}}{\partial P}\, dp = \left(\frac{\partial S_m}{\partial M}\right)_M \int_{P_o}^{P_s} \left(\frac{\partial T}{\partial P}\right)_M dp$$

$$M\left(r_s^{-2} - r_o^{-2}\right) = \left(\frac{\partial S_m}{\partial M}\right)_M (T_s - T_o)$$

Let's consider $M = vr + \frac{1}{2} f r^2$

In an intense hurricane $vr \gg \frac{1}{2} f r^2$

"cyclostrophic balance" in the eyewall

$$M \simeq vr$$

$$M_o = M_s$$

$$r_s v_s = r_o v_o \qquad r_o \gg r_s$$

We can use M definition to obtain:

$$\frac{v_s}{r_s} - \cancel{\frac{v_o}{r_o}} = \left(\frac{\partial S_m}{\partial M}\right)_M (T_s - T_o)$$

$$\frac{v_s}{r_s} = \left(\frac{\partial S_m}{\partial M}\right)_M (T_s - T_o)$$

$$\frac{\partial S_m}{\partial M} = \frac{DS_m/Dt}{DM/Dt} = \frac{\cancel{\dot{Q}_r} + L_v E + SHF + \boxed{v F_r}}{r F_r T_s}$$

frictional heating

$$= \frac{C_h V (h^* - h) + C_d V^3}{r C_d V^2 T_s} \qquad \frac{DS_m}{Dt} = \frac{\dot{Q}_e}{T}$$

$$\frac{V_s}{\cancel{r_s}} = \frac{(T_s - T_o)}{T_s} \frac{C_h V (h^* - h) + C_d V^3}{T_s \cancel{r_s} C_d V^2}$$

$$C_d V^3 = \frac{(T_s - T_o)}{T_s}\left[C_h \cancel{V} (h^* - h) + C_d V^3\right]$$

Moving things around yields

$$V_{max}^2 = \frac{C_h}{C_d} \frac{T_s - T_o}{T_o} (h^* - h)$$

Max. Pot. Intensity