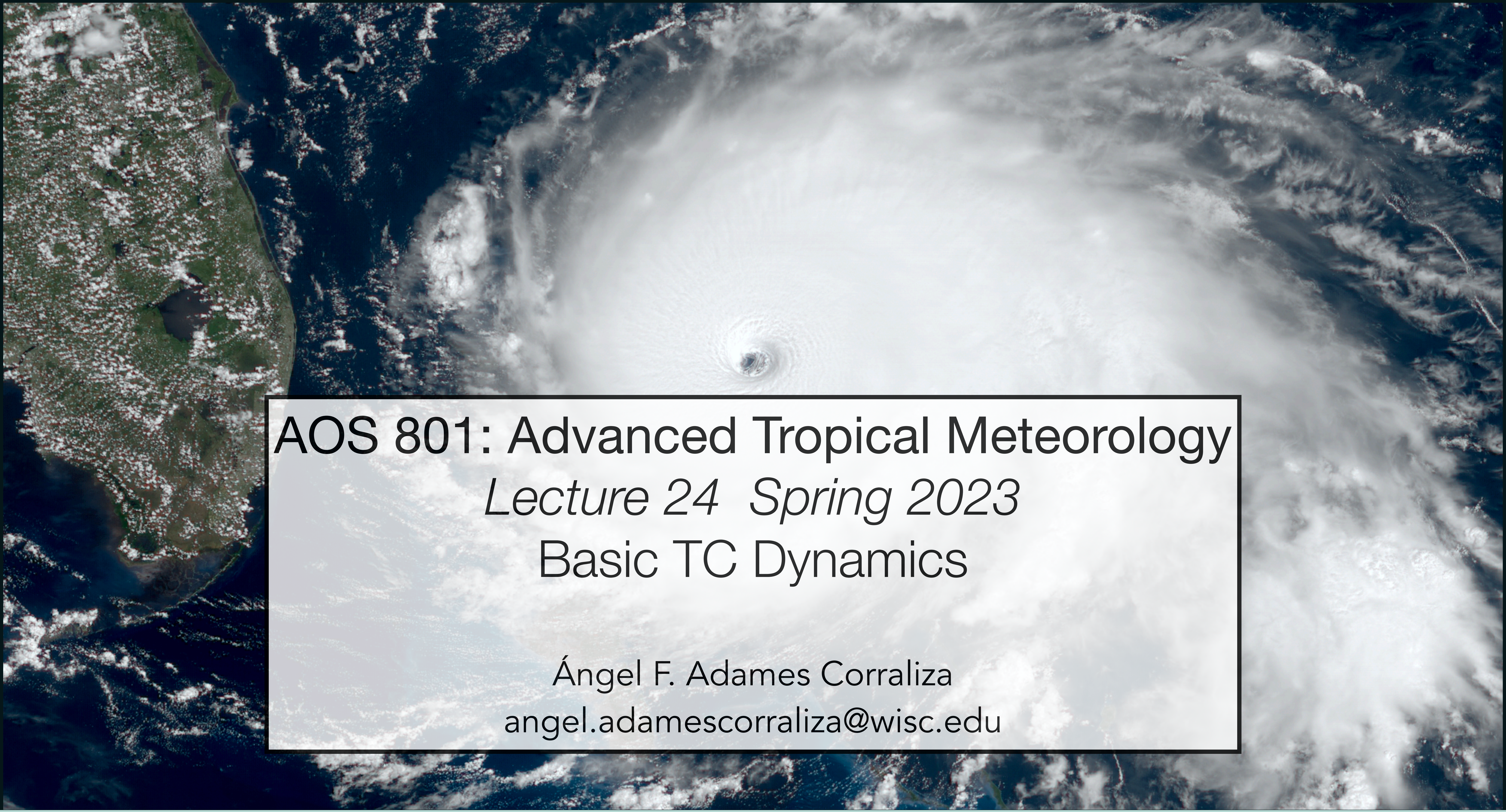

AOS 801: Advanced Tropical Meteorology

Lecture 24 Spring 2023

Basic TC Dynamics

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Announcements

HW5 and PA5 are due next Tuesday.

Guidelines to your final presentation are now on Canvas.

Final lectures are this week.

Gradient Balance

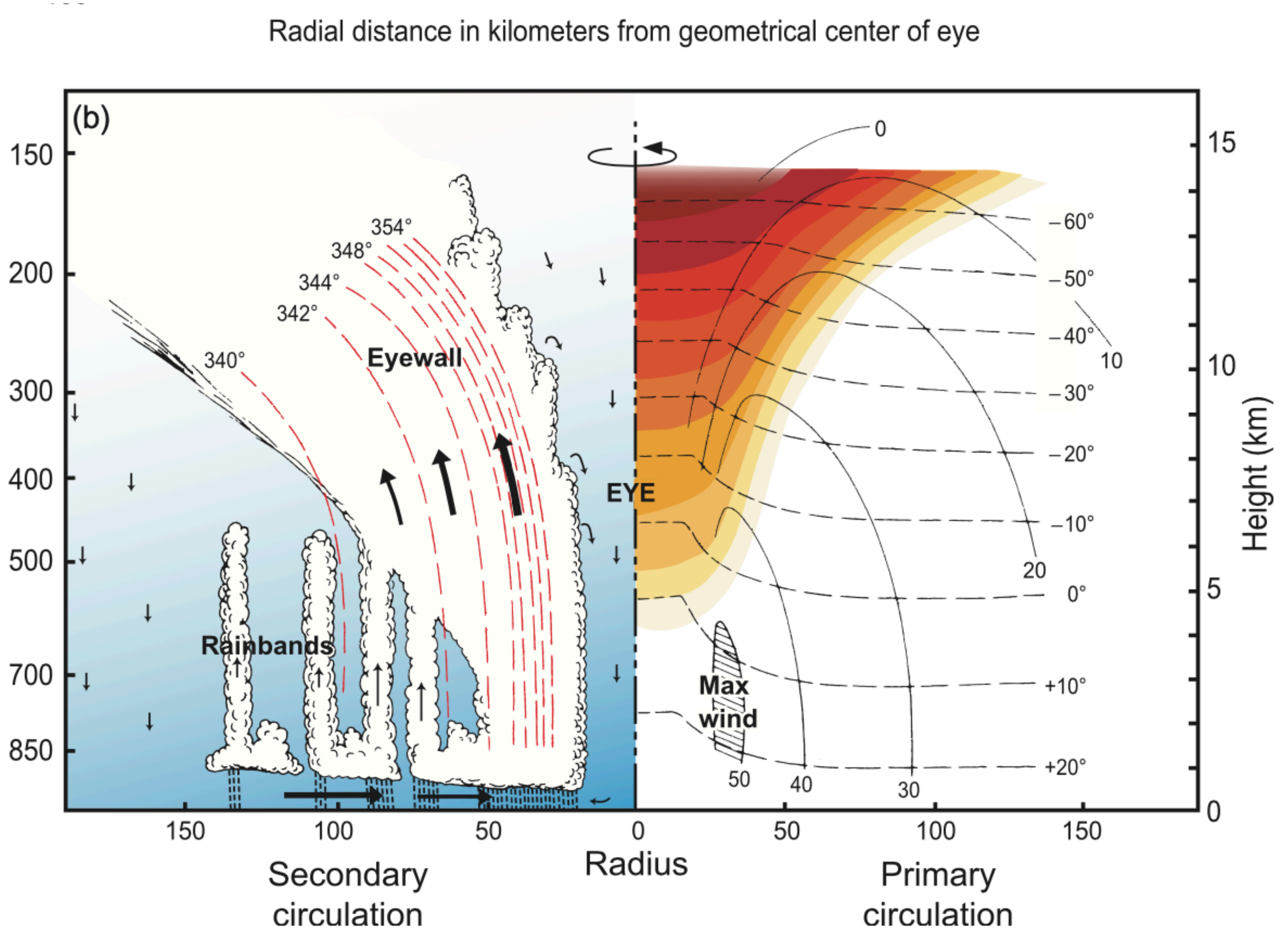
The tangential circulation is in gradient balance:

$$\frac{v^2}{r} + fv = \frac{\partial \Phi}{\partial r}$$

Using hydrostatic balance we can obtain the TC thermal wind:

$$\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial p} = -\frac{R_d}{p}\frac{\partial T}{\partial r}$$

Plugging some reasonable numbers yields $\delta T \sim 6K$. Hurricanes are major departures from WTG balance.



Houze (2014)

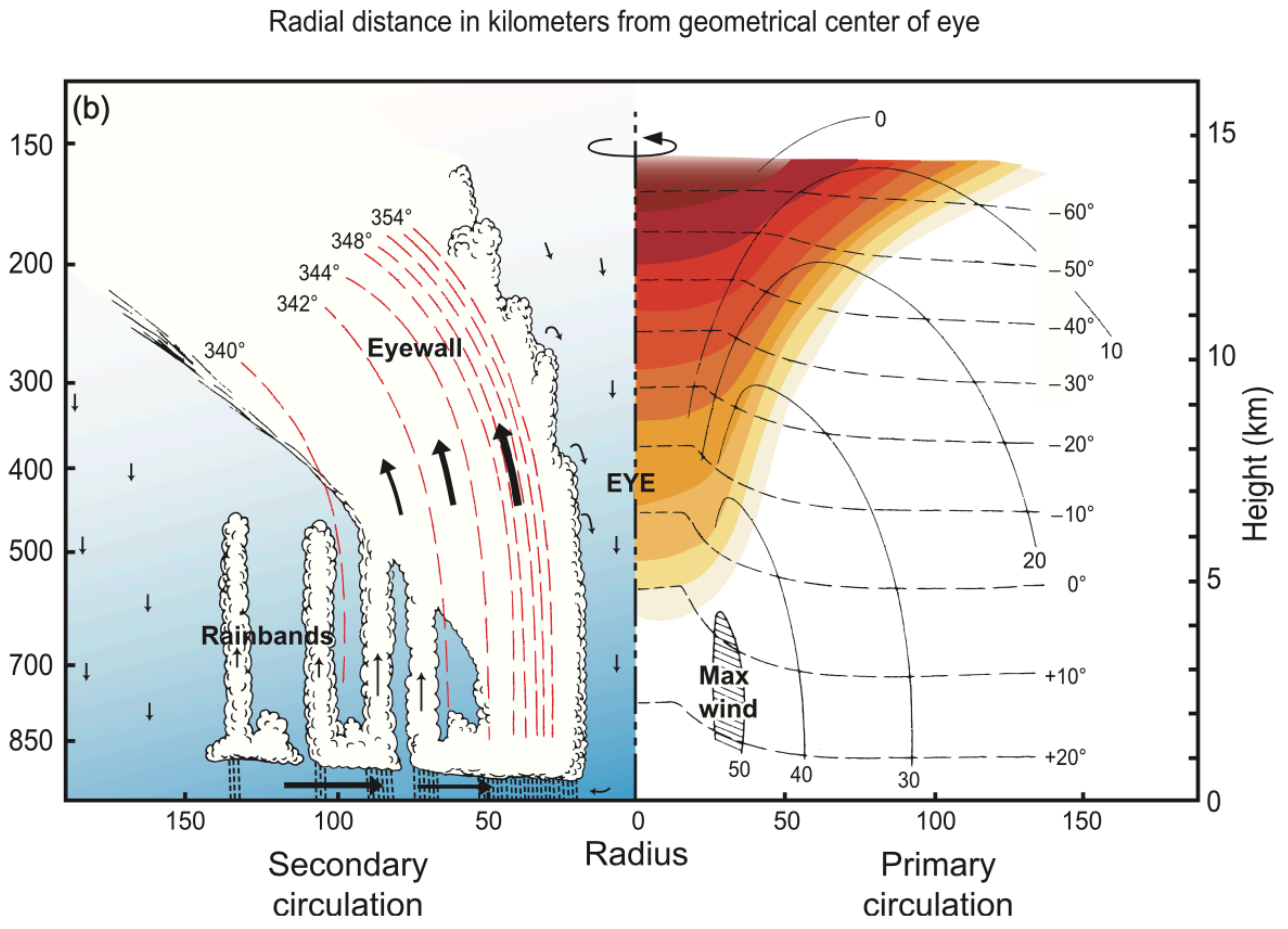
Gradient Balance

It is convenient to write the TC's tangential circulation in terms of "absolute angular momentum"

$$M = vr + \frac{1}{2} f r^2$$

So that the thermal wind equation becomes:

$$\frac{\partial M^2}{\partial p} = -\frac{R_d r^3}{p}\frac{\partial T}{\partial r}$$



Houze (2014)

The breakdown of WTG balance in the TC core

Let's consider the horizontal momentum equation in isobaric coordinates (for simplicity)

$$\frac{D\mathbf{v}}{Dt} = -\nabla_h \Phi - f\mathbf{k} \times \mathbf{v} - \mathbf{F}_r$$

We can use the vector identity for advection to re-express the horizontal momentum advection as

$$(\mathbf{v} \cdot \nabla_h)\mathbf{v} = \nabla_h \underbrace{\left(\frac{\mathbf{v} \cdot \mathbf{v}}{2}\right)}_{\text{KE}} + \zeta \mathbf{k} \times \mathbf{v}$$

Yielding:

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla_h(\Phi + \text{KE}) - \zeta_a \mathbf{k} \times \mathbf{v} - \omega \frac{\partial \mathbf{v}}{\partial p} - \mathbf{F}_r$$

The breakdown of WTG balance in the TC core

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla_h(\Phi + K) - \zeta_a \mathbf{k} \times \mathbf{v} - \omega \frac{\partial \mathbf{v}}{\partial p} - \mathbf{F}_r$$

Scale analysis of the equation yields

$$\mathrm{Ro}_{TC} = \frac{1}{\tau \zeta_a} \qquad L_d = \frac{c}{\zeta_a}$$

WTG scaling becomes

$$\mathrm{N}_w = \frac{L^2}{L_d^2}$$

If $\zeta_a \sim 5 \times 10^{-4}\ s^{-1}$, $c = 50\ m/s$ then $L_d \sim 100\ km$!

It is very difficult to have WTG in a mature TC because of the strong amount of rotation in the TC itself.

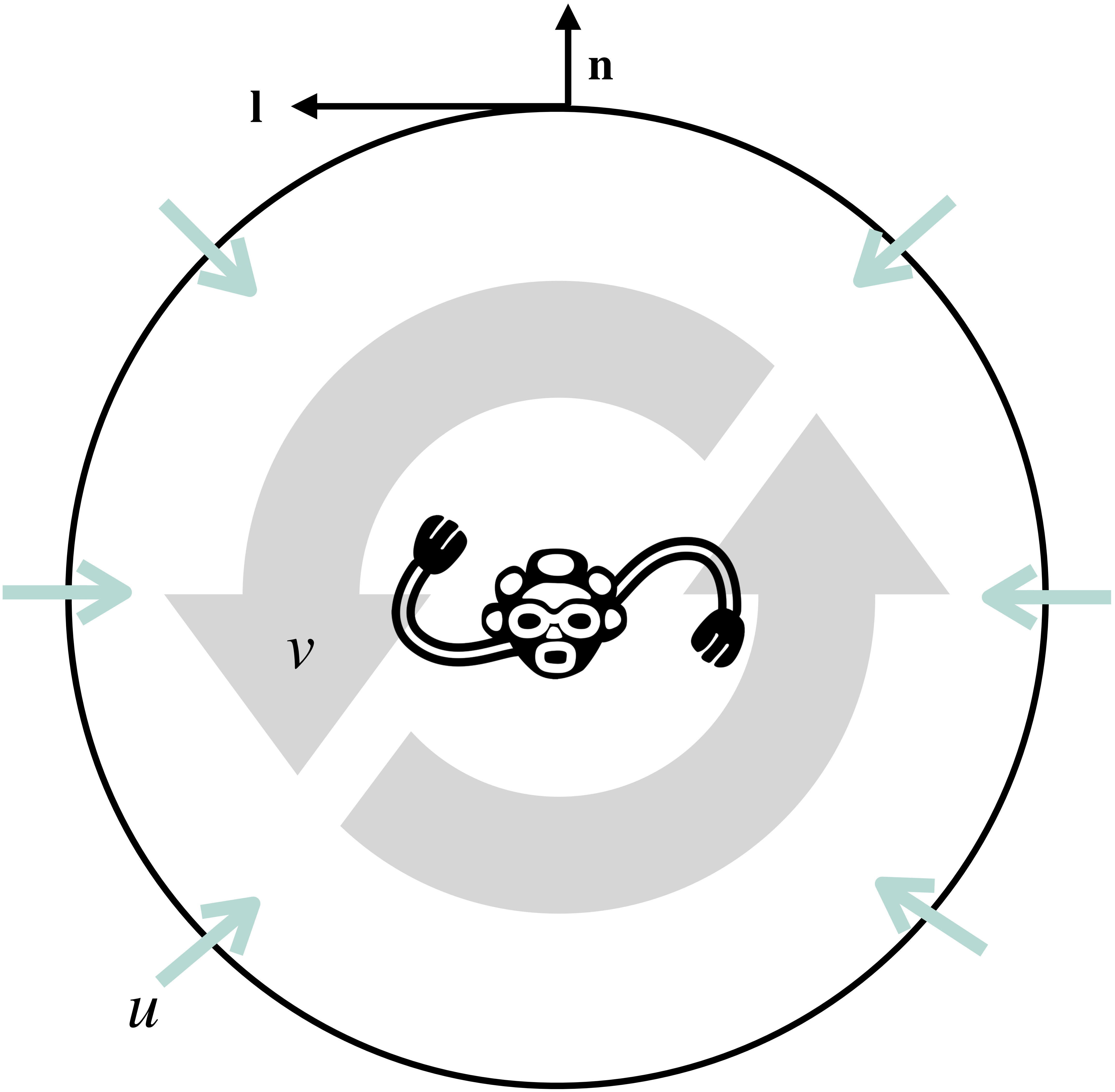
Does WTG balance allow for its breakdown?

Let's look at the WTG momentum equations derived previously, but including friction

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left(\mathbf{v}\zeta_a - \omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p} + \mathbf{k} \times \mathbf{F}_r \right)$$

$$\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\mathbf{v}\delta_w + \omega_w \frac{\partial \mathbf{v}}{\partial p} + \mathbf{F}_r \right) - \Sigma$$

While the TC itself may not be in WTG balance, we can use Stokes and the Divergence theorem to evaluate it at a distance where we can apply WTG.



Does WTG balance allow for its breakdown?

After application we get

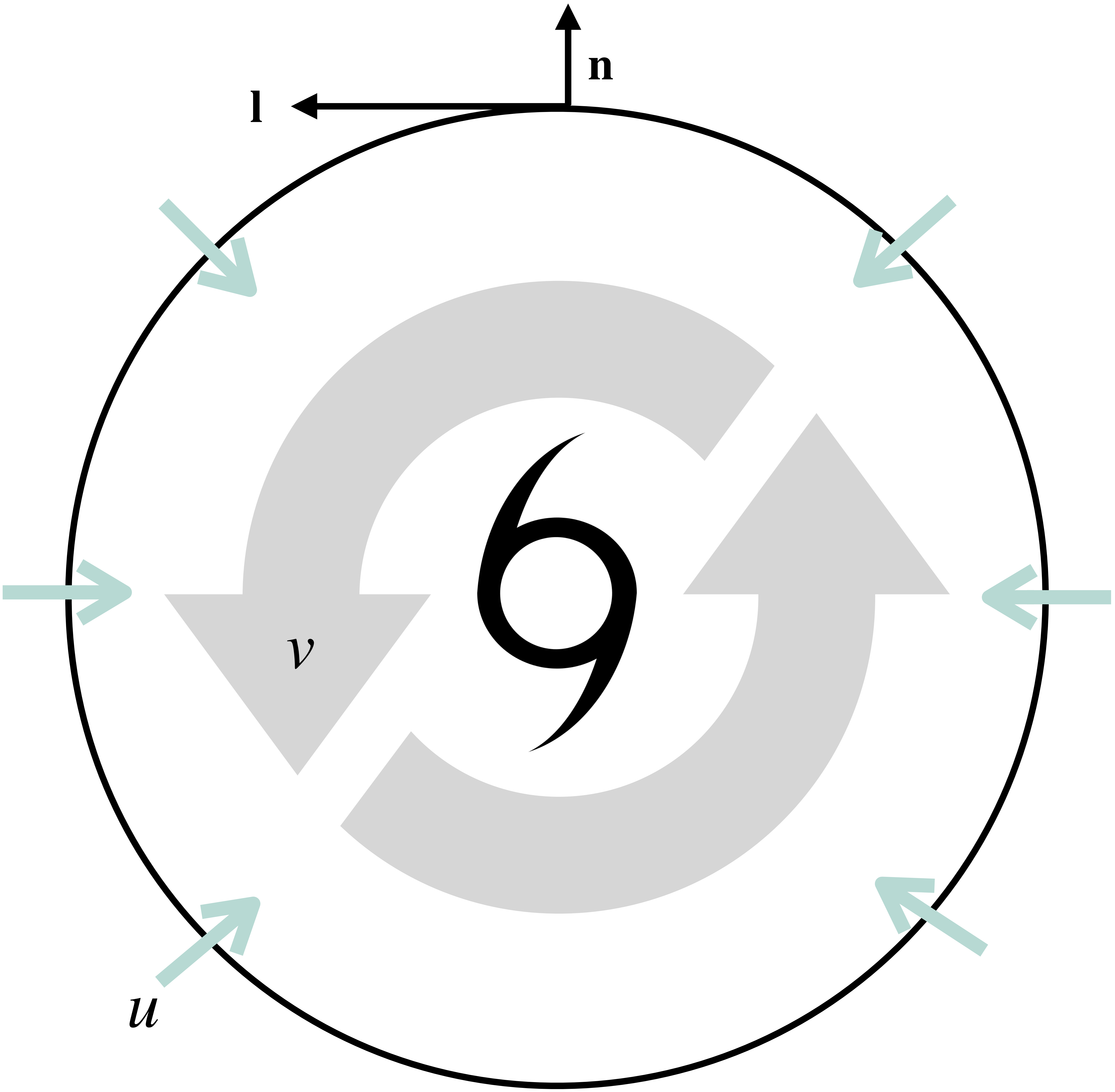
$$\frac{\partial v}{\partial t} = -u\zeta_a - F_\theta$$

$$\frac{\partial u}{\partial t} = -u\delta_w - F_r - [\Sigma]$$

Both equations are evaluated at radius R

So long as the inflow of absolute vorticity exceeds frictional dissipation, the vortex will strengthen

$$\left(\frac{2v}{r} + f\right)\frac{\partial v}{\partial p} = -\frac{R_d}{p}\frac{\partial T}{\partial r}$$



Does WTG balance allow for its breakdown?

After application we get

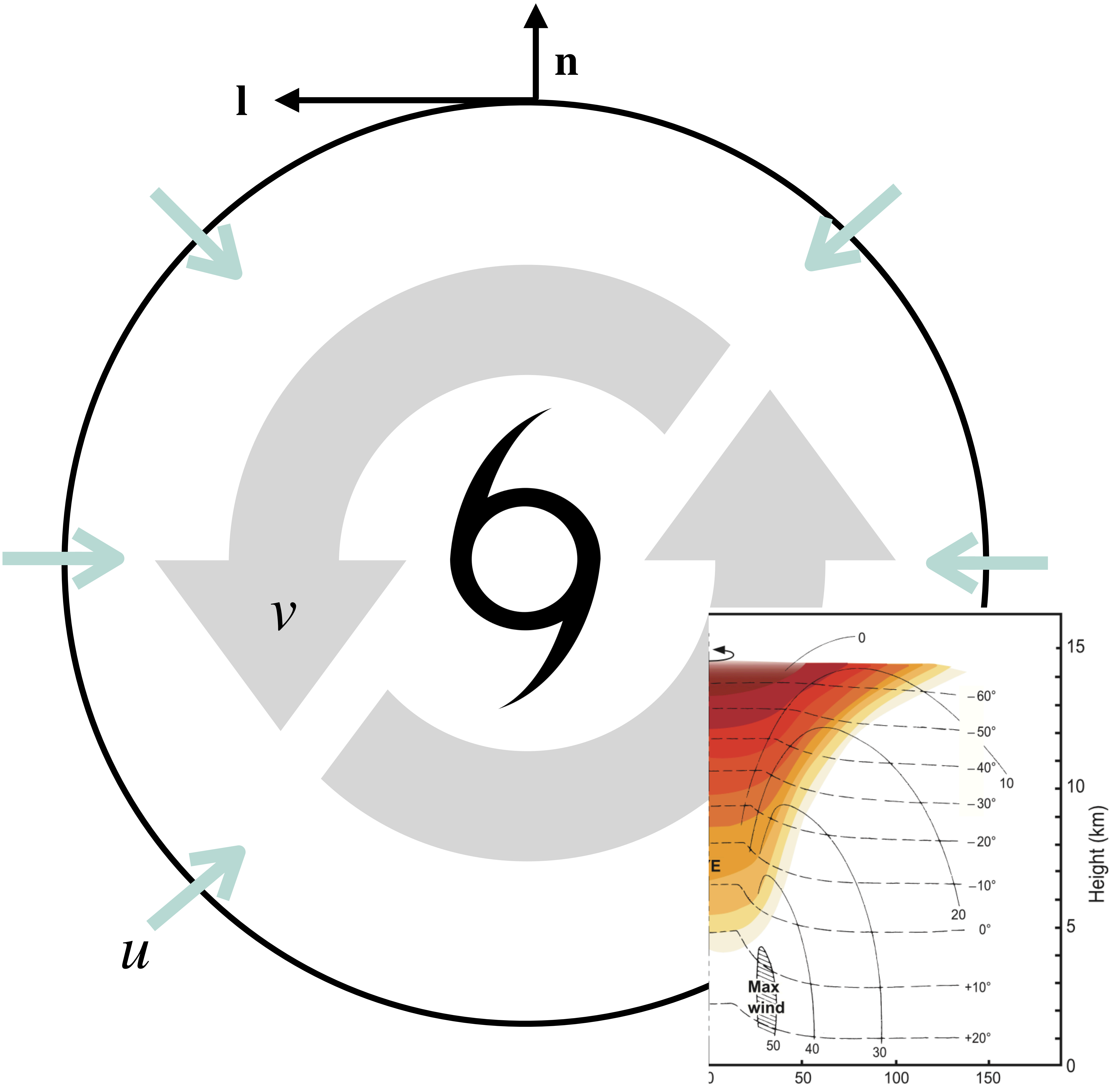
$$\frac{\partial M}{\partial t} = -u\frac{\partial M}{\partial r} - rF_\theta$$

$$\frac{\partial u}{\partial t} = -\frac{u}{r}\frac{\partial ru}{\partial r} - F_r - [\Sigma]$$

$$\frac{\partial M^2}{\partial p} = -\frac{R_d r^3}{p}\frac{\partial T}{\partial r}$$

Equations are evaluated at radius R

So long as the inflow of angular momentum exceeds frictional dissipation, the vortex will strengthen



Paper discussion

Get into groups and discuss the following questions:

1. What is barotropic instability and how does it compare to MVI?

2. Discuss Fig. 11

3. If you were to looking at regions for potential TC genesis, what would you look at based on the results of this study?

Paper discussion

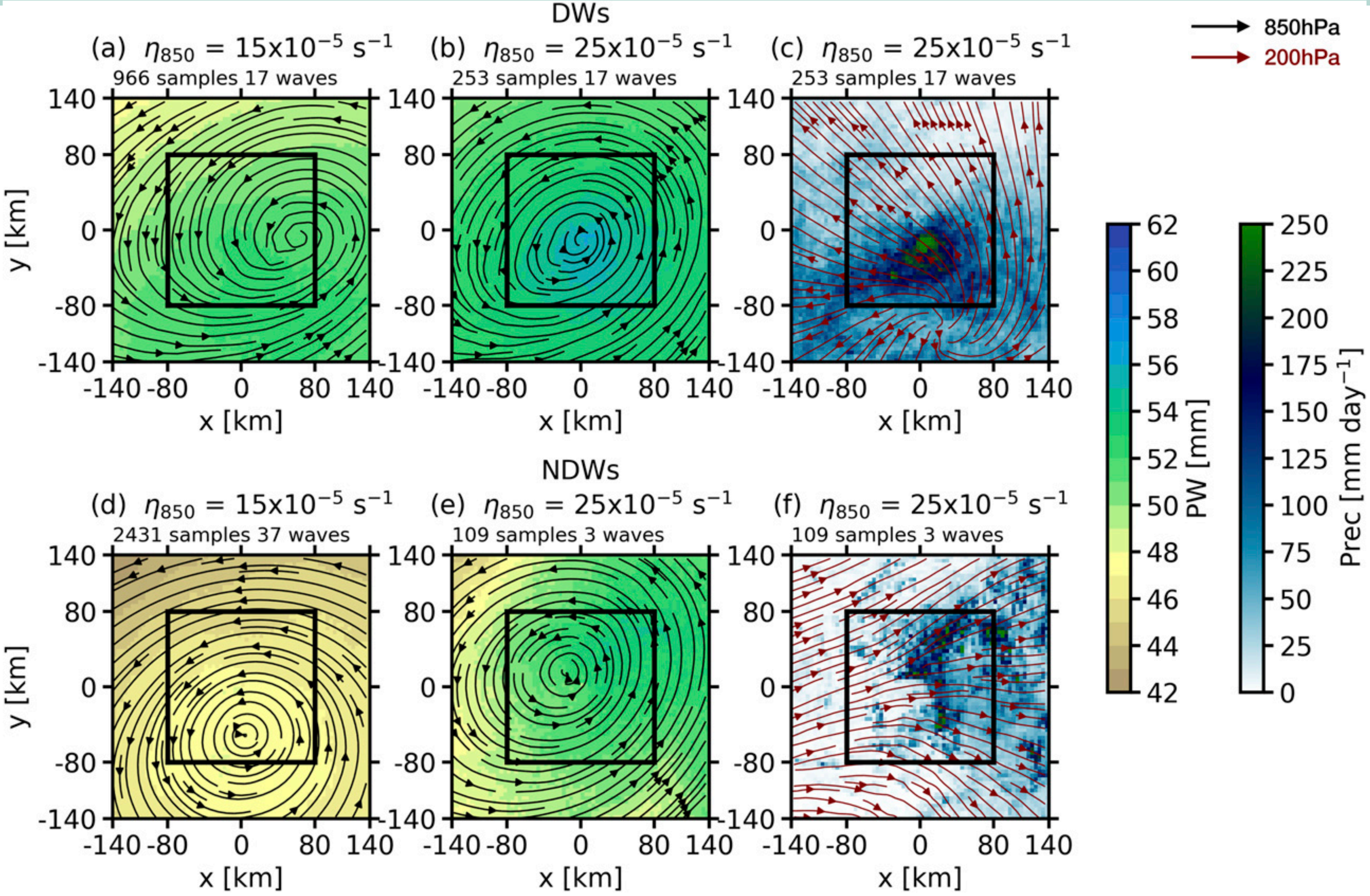


FIG. 11. Composites of hourly (left),(center) PW and (right) Prec binned by hourly averaged $160 \times 160\,\mathrm{km}^2$ blocks of η_{850} following the vortices, with the same binning approach as in Fig. 10. The PW composites are from 15×10^{-5} and $25 \times 10^{-5}\,\mathrm{s}^{-1}$ η_{850} bins, and the Prec composites are from the $25 \times 10^{-5}\,\mathrm{s}^{-1}$ η_{850} bin. Rows show the composites of (a)–(c) the developing waves (DWs) and (d)–(f) the nondeveloping waves (NDWs). The thin black (brown) lines with arrows overlaid on the PW (Prec) composites are the streamflows of horizontal winds at 850 hPa (200 hPa). The thick black lines indicate the $160 \times 160\,\mathrm{km}^2$ domain.