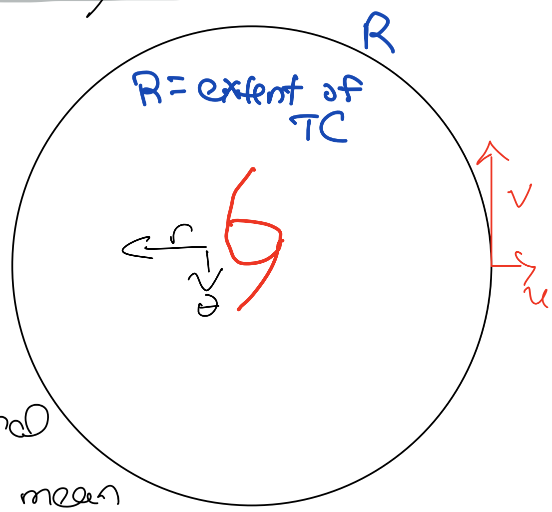
WTG breakdown in TCs

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left(\vec{v}\zeta_a - \cancel{\omega_w \hat{k} \times \frac{\partial \vec{v}}{\partial p}} + \hat{k} \times \vec{F_r} \right)$$ WTG vort.

$$\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\vec{v}\delta_w + \cancel{\omega_w \frac{\partial \vec{v}}{\partial p}} + \vec{F_r} \right) - \Sigma$$ WTG div



Assume that the entire TC is enclosed in a circle of radius r

× We can average the area of the circle to know how the TC evolves. Let's assume weak vertical shear, and that we follow the mean flow

$$\frac{1}{2\pi R}\int_\theta\int_r \frac{\partial \zeta}{\partial t}\, dr\, d\theta \qquad \zeta = \hat{k}\cdot(\nabla \times \vec{v})$$

$$\frac{1}{2\pi R}\int_\theta\int_r \frac{\partial}{\partial t}\left(\hat{k}\cdot(\nabla \times \vec{v})\right) dr\, d\theta$$

$$\frac{1}{2\pi R}\frac{\partial}{\partial t}\int_\theta\int_r \left(\hat{k}\cdot \nabla \times \vec{v}\right) dr\, d\theta$$

Stokes theorem says that

$$\int_\theta\int_r \hat{k}\cdot(\nabla \times \vec{v})\, dr\, d\theta = \oint_\theta \vec{v}\cdot d\vec{\ell}$$

$$= \oint_\theta v\, d\ell = \int_0^{2\pi} vR\, d\theta$$

$$\frac{1}{2\pi R}\int_\theta\int_r \frac{\partial \zeta}{\partial t}\, dr\, d\theta = \left.\frac{\partial v_t}{\partial t}\right|_R$$

We can apply the divergence theorem on the rhs, and rinse and repeat for div equation (note: S uses divergence theorem)