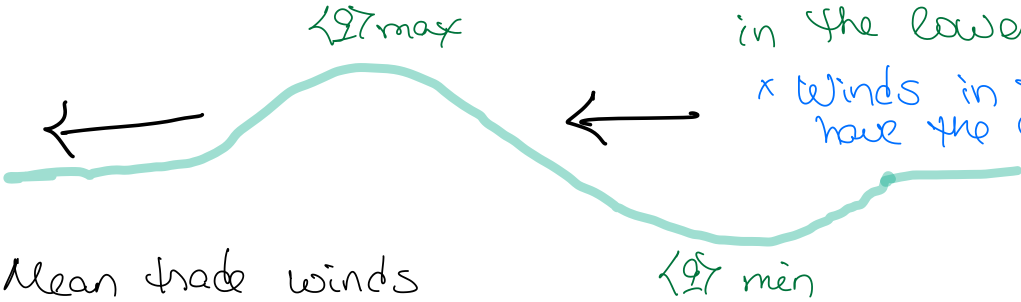
From HW4, you obtain a set of equations for TD-like waves:

$$\frac{\partial \zeta_L'}{\partial t} \simeq -\bar{u}_L \frac{\partial \zeta_L'}{\partial x} + \frac{f_0 L_v \langle q' \rangle}{S T_c}$$

$$\frac{\partial \langle q' \rangle}{\partial t} = -\bar{u}_L \frac{\partial \langle q' \rangle}{\partial x} - v_L' \frac{\partial \langle \bar{q} \rangle}{\partial y}$$

* Moisture is largest in the lower troposphere

* winds in the low trop. have the largest influence on moisture



Mean trade winds advect moisture westward.

Let's simplify by defining $\frac{D_h}{Dt} = \frac{\partial}{\partial t} + \bar{u}_L \frac{\partial}{\partial x}$

$$\frac{D_h \zeta_L'}{Dt} = \frac{f_0 L_v \langle q' \rangle}{S T_c}$$ vortex stretching

$$\frac{D_h \langle q' \rangle}{Dt} = - v_L' \frac{\partial \langle \bar{q} \rangle}{\partial y}$$ adv. of mean moisture by anom. meridional winds

Let's follow Sobel et al. (2001) and assume that

$$\frac{\omega - \bar{u}_L k}{f_0} \ll 1$$ Why?

freq

doppler shift

planetary vort. "const"

When $\omega - \bar{u}_L k \ll f_0$ the hor. winds are predominantly non-divergent.

$$v' \simeq \frac{\partial \psi'}{\partial x} \quad \text{streamfunction} \qquad \xi' = \nabla_h^2 \psi$$

Replacing with ψ:

$$\frac{D_h \nabla_h^2 \psi'}{Dt} = \frac{f_0 L_v \langle q' \rangle}{S \tau_c} \quad (1)$$

$$\frac{D_h \langle q' \rangle}{Dt} = -\frac{\partial \psi'}{\partial x} \frac{\partial \langle \bar{q} \rangle}{\partial y}$$

We have two eqns. with two variables: ψ and $\langle \xi' \rangle$

Assuming that $\bar{u}_L$, $\frac{\partial \langle \bar{q} \rangle}{\partial y}$, f_0, S, and τ_c are constants, then solutions to the pair of eqns. takes the form:

$$\psi(x,y,t) = \psi_0 \exp(ikx + ily - i\omega t)$$
$$\langle q \rangle (x,y,t) = \langle q_0 \rangle \exp(ikx + ily - i\omega t)$$

Let's define $\omega^* = \omega - \bar{u}_L k$ as the doppler-shifted freq.

$$\nabla_h^2 \psi' = -(k^2 + l^2)\psi' \qquad \frac{\partial \psi'}{\partial x} = ik\psi$$

$$\frac{D_h}{Dt} \rightarrow -i\omega^*$$

Replacing on Eq. (1) gives:

$$-i\omega^*(-k^2 - l^2)\psi' = \frac{f_0 L_v \langle q' \rangle}{S \tau_c}$$

$$-i\omega^* \langle q' \rangle = -ik\psi \frac{\partial \langle \bar{q} \rangle}{\partial y}$$

We have removed the derivatives, and the resulting eqs. are solved by substitution

By doing that you obtain the dispersion:

$$\boxed{\omega = \bar{u}_L k \pm \sqrt{\frac{i\beta_g k}{K^2 \tau_c}}} \qquad K^2 = k^2 + \ell^2$$

Dispersion relation

$\omega = \omega_r + i\omega_i$

real — describes propagation

imag. — describes growth/decay

$\omega_r > 0$ eastward
$\omega_r < 0$ westward
$\omega_i > 0$ growth
$\omega_i < 0$ decay

$$\sqrt{i} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

westward, eastward

$$\omega_r = \bar{u}_L k \pm \frac{1}{2}\sqrt{\frac{\beta_g k}{K^2 \tau_c}}$$

$\bar{u}_L < 0$ westward prop

$$\omega_i = \pm \frac{1}{2}\sqrt{\frac{\beta_g k}{K^2 \tau_c}}$$

growth

When using the (+) solution, the wave is unstable and grows.

In moisture modes $Ro \sim 1$

$= \frac{U}{f_0 L} \sim 1$

$\frac{\bar{u}k}{f_0} \sim 1$

But recall that $\omega - \bar{u}k \ll f_0$

$$\pm\sqrt{\frac{\beta_g k}{K^2 \tau_c}} \ll f_0$$

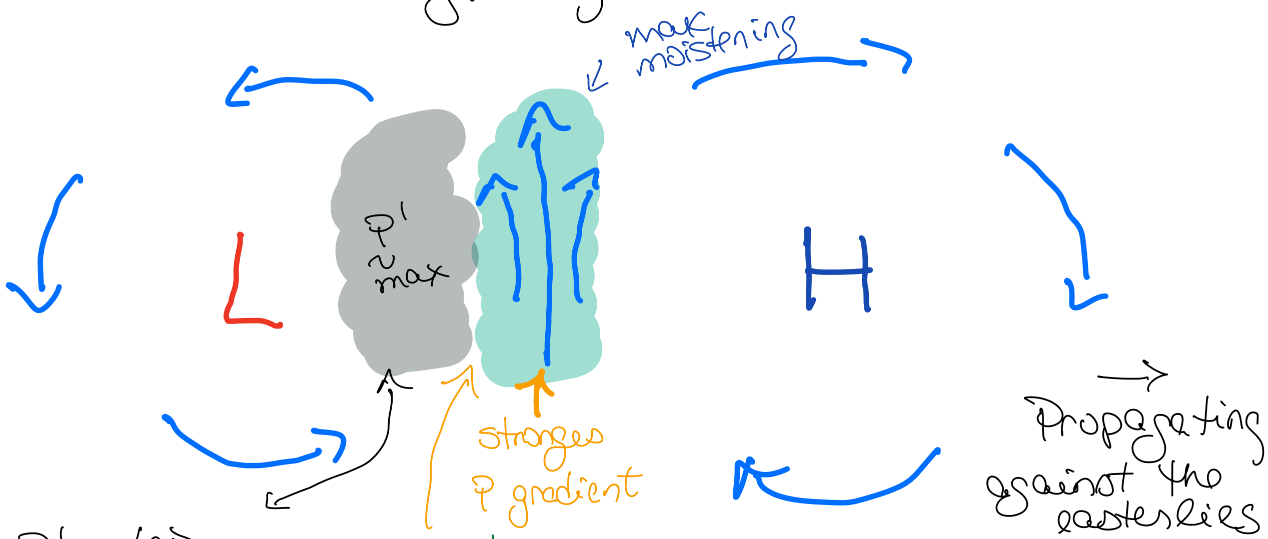
which means that if $\bar{u}k \sim f_0$ then

$$\omega_r = \bar{u}_L k \pm \frac{1}{2}\sqrt{\frac{\beta_g k}{K^2 \tau_c}} \simeq \bar{u}_L k$$

$\gg$

wave propagates westward (albeit a little more slowly than $\bar{u}_L$)

* Solution shows growing wave that moves west.



$P' = \frac{\langle q \rangle}{\tau_c}$

$v' \sim max$

$-v' \frac{\partial \langle q \rangle}{\partial y} \sim max \rightarrow \frac{D_h \langle q \rangle}{Dt} \sim max$

x wind anomalies are moistening the column via moisture advection!

x The max moisture does not occur where $\partial q / \partial t$ is a max. It occurs afterwards.

$$\zeta_L' \frac{D_h \zeta_L'}{Dt} = -f_0 \delta \zeta_L' \qquad \frac{1}{2} \frac{D_h \zeta_L'^2}{Dt} = -f_0 \delta_w' \zeta'$$