Moisture variance under WTG balance (moisture modes)

When $N_{mode} \ll 1$ $\quad (C_pT' \ll L_v q')$

$$\frac{\partial \langle MSE \rangle}{\partial t} \simeq \frac{\partial L_v \langle q \rangle}{\partial t}$$

$$\frac{1}{2}\frac{\partial L_v \langle q \rangle^2}{\partial t} = \langle q \rangle \left(-\langle \vec{v} \cdot \nabla L_v q \rangle - \left\langle \omega_w \frac{\partial MSE}{\partial p} \right\rangle + \langle Q_r \rangle + L_v E + SHF \right)$$

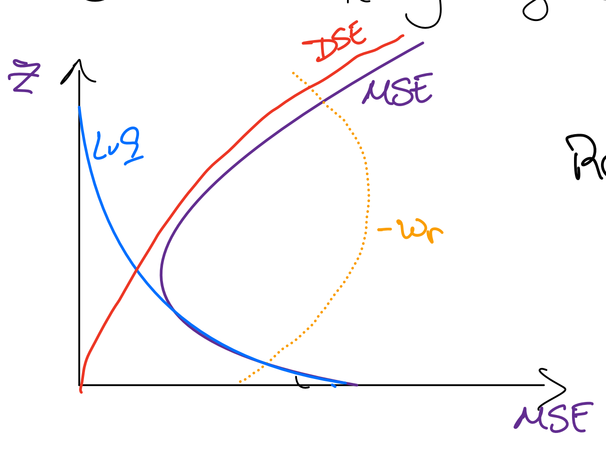
Recall that: $\omega_w = \omega_c + \omega_r \qquad \omega_r \frac{\partial DSE}{\partial p} = Q_r$

$$\left\langle \omega_w \frac{\partial MSE}{\partial p} \right\rangle = \left\langle (\omega_c + \omega_r) \frac{\partial MSE}{\partial p} \right\rangle \qquad \omega_c \gg \omega_r$$

$$\simeq \left\langle \omega_c \frac{\partial MSE}{\partial p} \right\rangle$$

$$\langle Q_r \rangle \simeq \left\langle \omega_r \frac{\partial DSE}{\partial p} \right\rangle \simeq -\left\langle \omega_r \frac{\partial L_v q}{\partial p} \right\rangle$$

Because in rainy regions $\left\langle \omega_r \frac{\partial MSE}{\partial p} \right\rangle \ll \left\langle \omega_r \frac{\partial L_v q}{\partial p} \right\rangle$



Recall: $\left\langle \omega_r \frac{\partial MSE}{\partial p} \right\rangle$ ← smaller than individual terms

$$= \left\langle \omega_r \frac{\partial DSE}{\partial p} \right\rangle + \left\langle \omega_r \frac{\partial L_v q}{\partial p} \right\rangle$$

$$= \langle Q_r \rangle + \left\langle \omega_r \frac{\partial L_v q}{\partial p} \right\rangle$$

roughly cancel

$$\langle Q_r \rangle \sim -\left\langle \omega_r \frac{\partial L_v q}{\partial p} \right\rangle$$

Instability in the column

Inoue and Back (2017), Inoue et. al (2021)

$$\frac{\partial L_v\langle q\rangle}{\partial t} = -\Gamma_e \nabla\cdot\langle\vec{v}DSE\rangle \qquad \text{WTG}$$

$$\nabla_h\cdot\langle\vec{v}DSE\rangle = \langle Q_1\rangle$$
$$= L_v P + \langle Q_r\rangle$$

Let's assume that $P' \approx \dfrac{\langle q'\rangle}{\tau_c}$ ← conv. moisture adj. timescale

$\tau_c \simeq$ time you need for P to remove $\langle q\rangle$.

If we linearize the system, we can show that (qualitatively)

$$\cancel{L_v}\,\tau_c\frac{\partial P'}{\partial t} \simeq -\bar{\Gamma}_e \cancel{L_v} P \qquad \Gamma_e \simeq \text{complex constant}$$

$$\frac{\partial P'}{\partial t} = \frac{-\bar{\Gamma}_e P'}{\tau_c} \qquad \Gamma_e \text{ and } \tau_c \text{ are constants for the sake of argument.}$$

Solving this equation yields

$$P'(t) = P_0 \exp\left(-\frac{\bar{\Gamma}_e}{\tau_c}t\right)$$

Assumming Γ_e is real then P either exponentially increases or decreases depending on sign of Γ_e

If we follow HW3 and say that clear sky radiation approx. cancels LE + SHF then we have that:

$$\Gamma_e = \frac{\langle \omega_c\,\partial_p MSE + \omega_r\,\partial_p L_v q\rangle}{\nabla_h\cdot\langle\vec{v}DSE\rangle} \simeq \underset{\text{GMS}}{\Gamma} - \underset{\text{radiation contr.}}{r(1-\Gamma)}$$

If $\Gamma_e < 0$ you get growth if $\Gamma_e > 0$ decay

In first baroclinic conv. $\Gamma > 0$ $\qquad r > 0$

for $\Gamma_e < 0$ $\qquad \Gamma < r(1-\Gamma)$ $\qquad$ if Γ is small

$$\Gamma < r$$

GMS $\qquad$ radiation cont