



AOS 801: Advanced Tropical Meteorology  
*Lecture 16 Spring 2023*  
Moisture Modes

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# Announcements

Paper discussion is on Wednesday

HW3 and PA3 are due on April 6.

While the motion is restricted to evolve in the horizontal plane, the isobars are still coupled:

$$\frac{\partial \zeta}{\partial t} = - \nabla_h \cdot \left( \mathbf{v} \zeta_a - \omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p} \right)$$

$$\frac{\partial \delta_w}{\partial t} = - \nabla_h \cdot \left( \mathbf{v} \delta_w + \omega_w \frac{\partial \mathbf{v}}{\partial p} \right) - \Sigma$$

# Lagrangian view on WTG balance

While the motion is restricted to evolve in the horizontal plane, the isobars are still coupled:

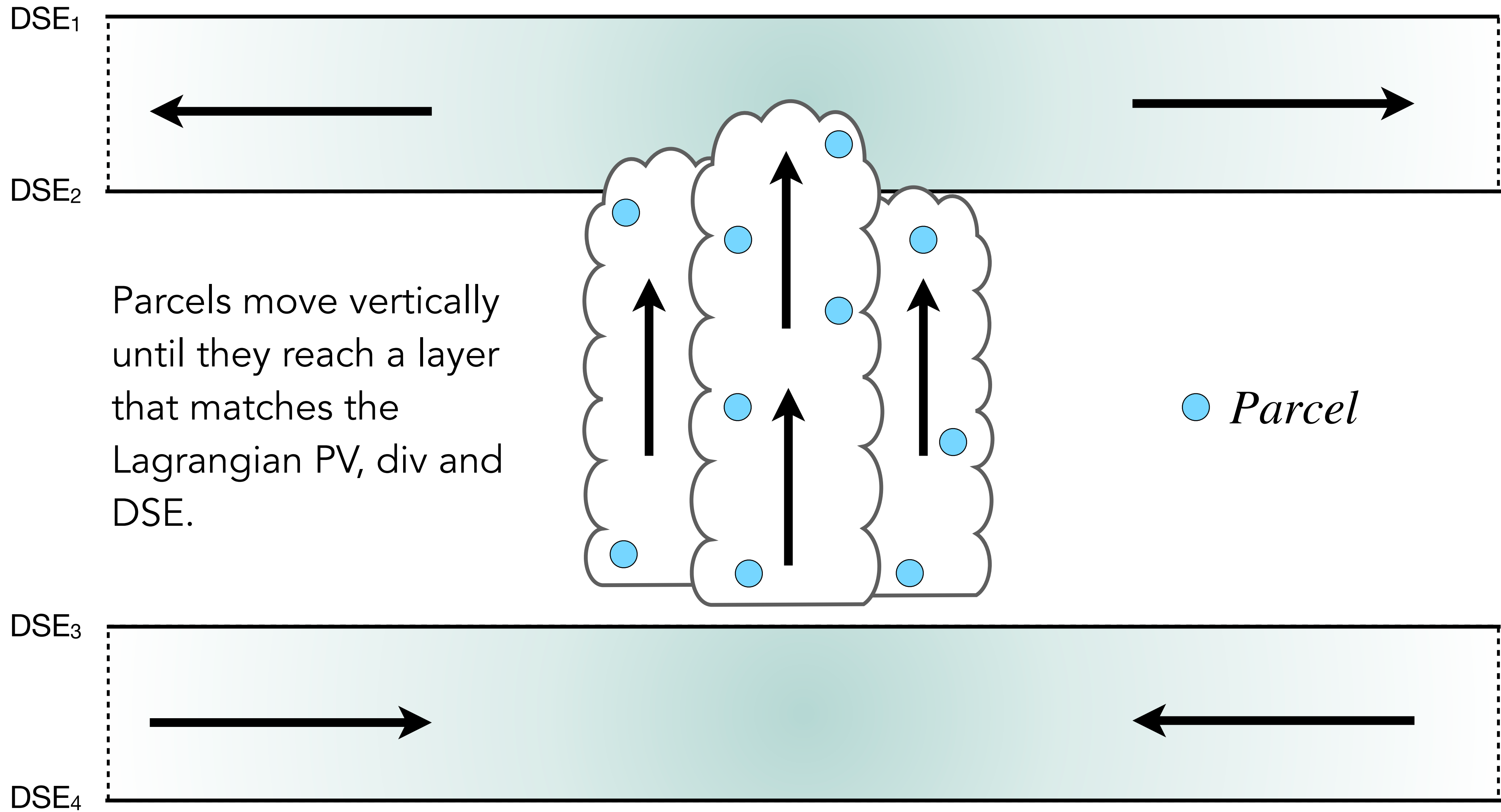
$$\frac{DPV}{Dt} = \nabla \cdot \eta_a Q_1$$

$$\frac{D\delta_w}{Dt} = -\Sigma + \nabla \cdot \frac{\partial \mathbf{u}}{\partial p} \frac{Q_1}{S_p}$$

$$\frac{DDSE}{Dt} = Q_1$$

The three dimensional structure then tells you how

# How to reconcile?



# Sticky Convection under WTG balance

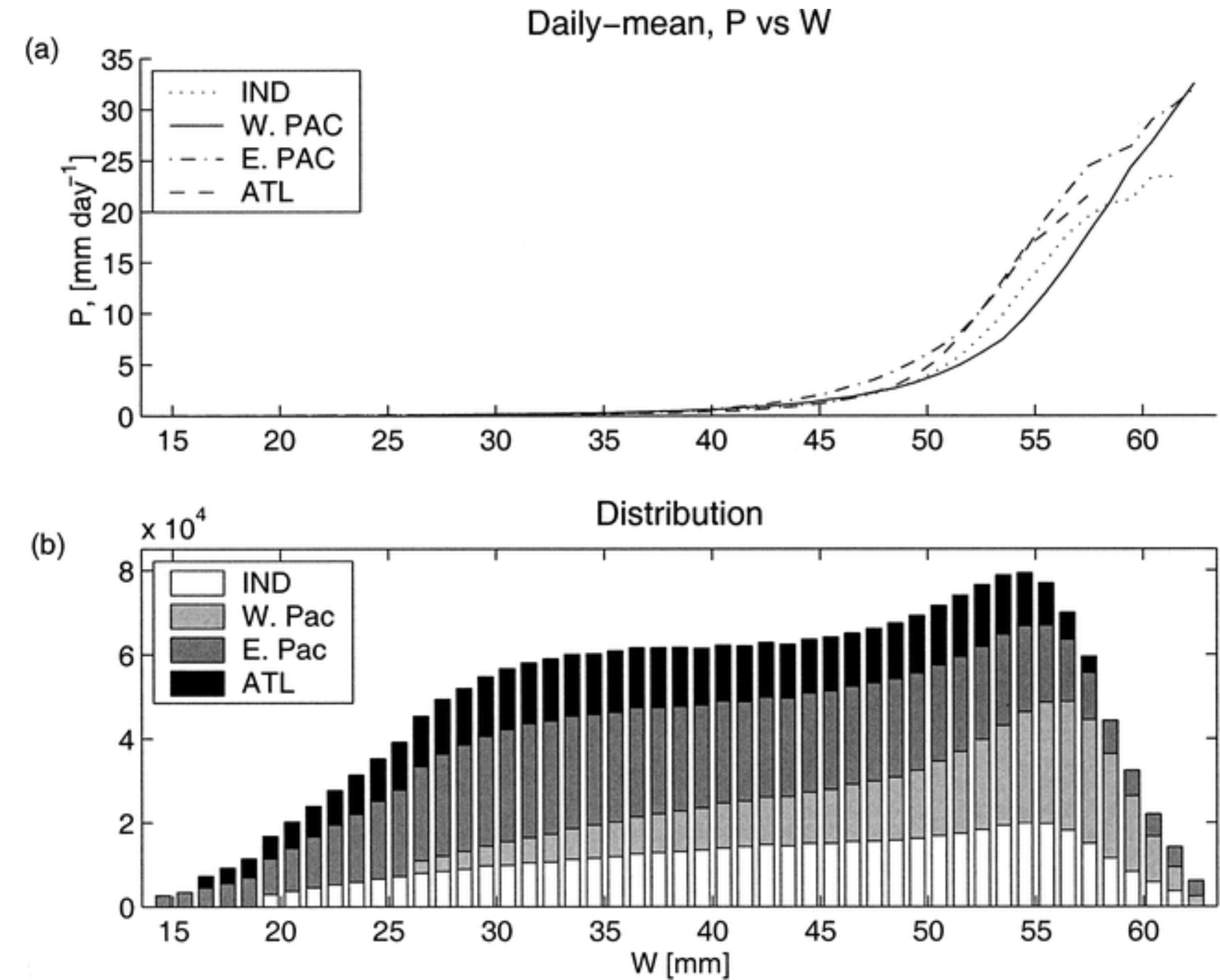
In L10 we found that:

$$L_v P = \frac{S_p L_v}{\mu_c^* m^4 \Delta p} \ln \left( \frac{T_{fl}}{T_{lnb}} \right) \nabla_h^2 \langle q \rangle - \langle Q_r \rangle$$

$$-\mathcal{O} \nabla_h^2 \langle q \rangle = L_v P + \langle Q_r \rangle$$

Under WTG we can show that moisture acts as a velocity potential, we have

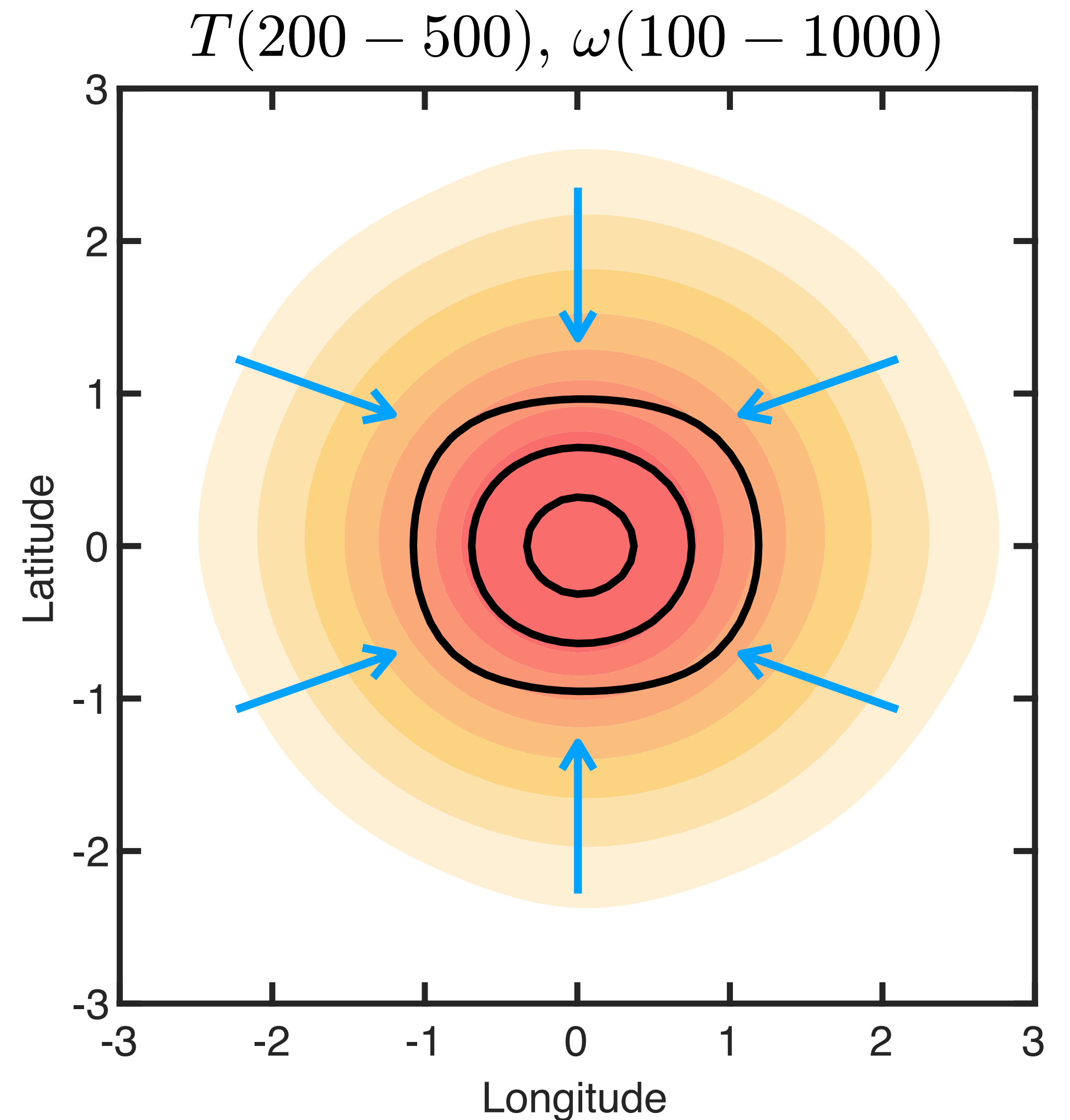
$$\mathcal{O} \nabla_h^2 \langle q \rangle = \nabla_h^2 \langle X_w \rangle S_p$$



# Buoyancy/Moisture as a velocity potential

In mesoscale regions of precipitation, we can interpret buoyancy/moisture as a velocity potential that satisfies

$$\langle X_w \rangle = \frac{\mathcal{O}}{S_p} \langle q \rangle$$



# Boundary layer quasi-equilibrium

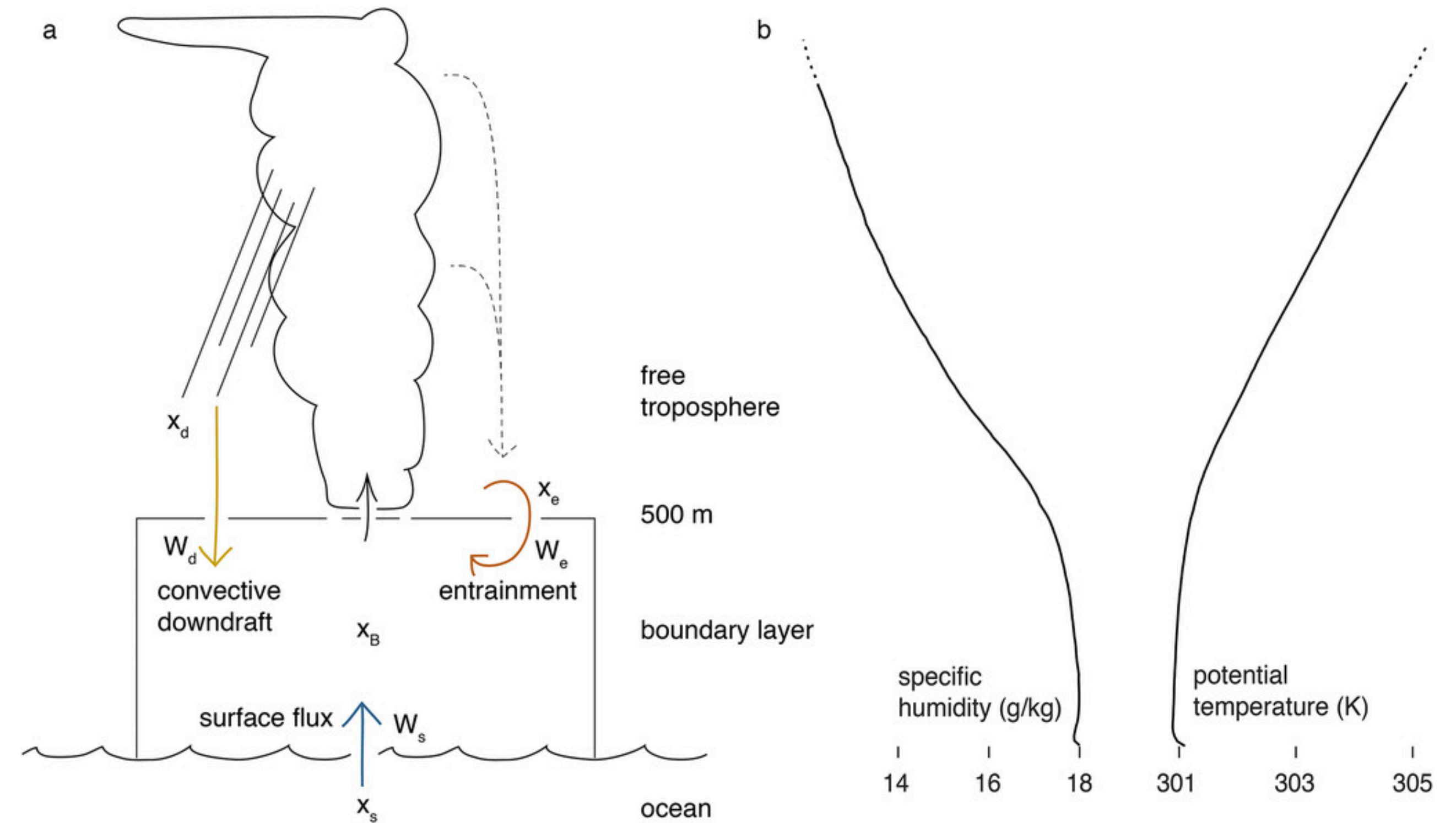
Lagrangian perspective on WTG balance

$$\omega_u = \frac{g (L_v E + \text{SHF})}{L_v \epsilon_p q_L^+}$$

Which we can write as

$$\nabla_h^2 X_w = \frac{g (L_v E + \text{SHF})}{L_v s_d q_L^+}$$

where  $s_d$  is the downdraft strength and  $q_L^+ = q_L^* - q_L$  is the saturation deficit

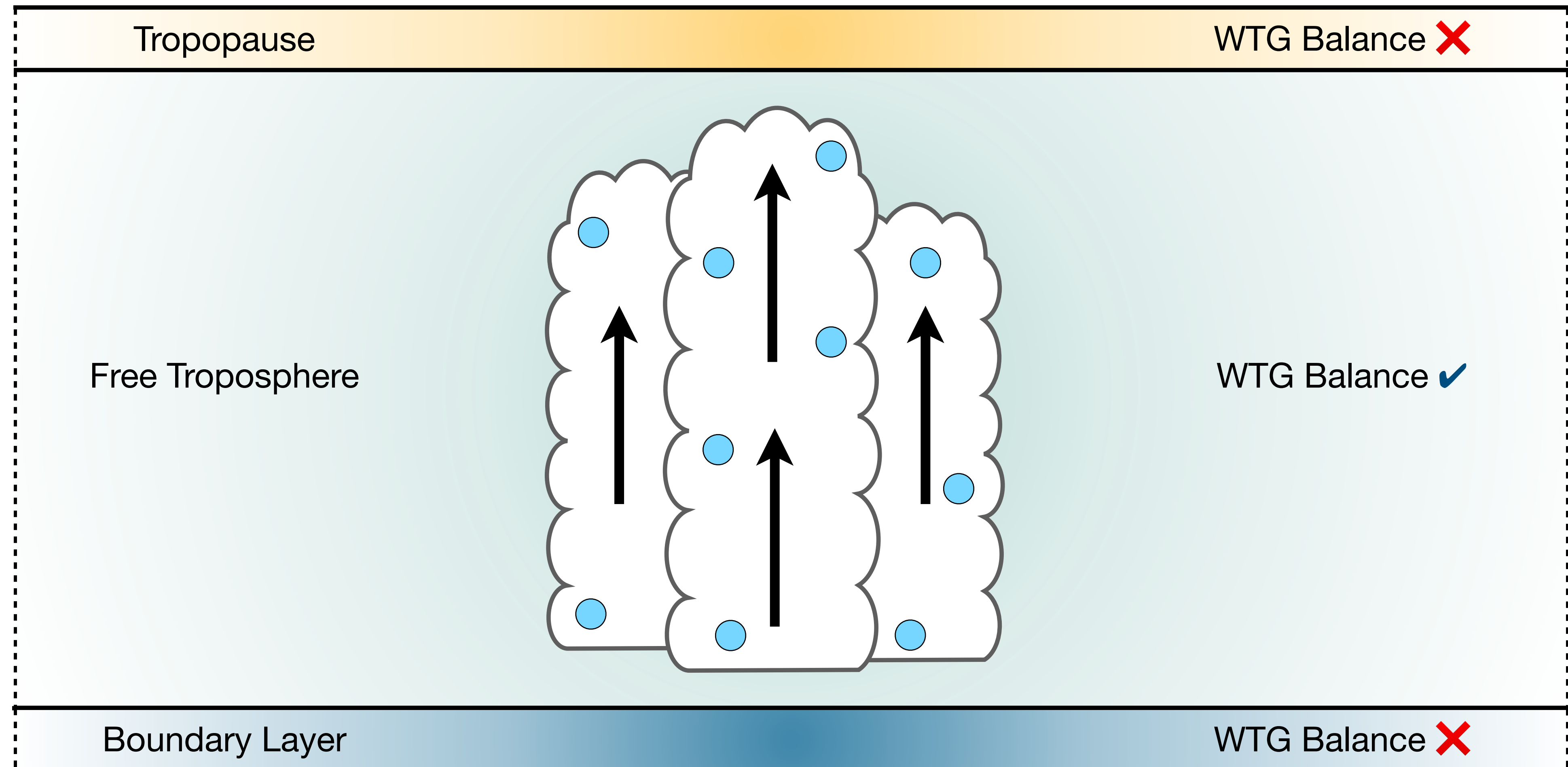


De Szoeke (2018)

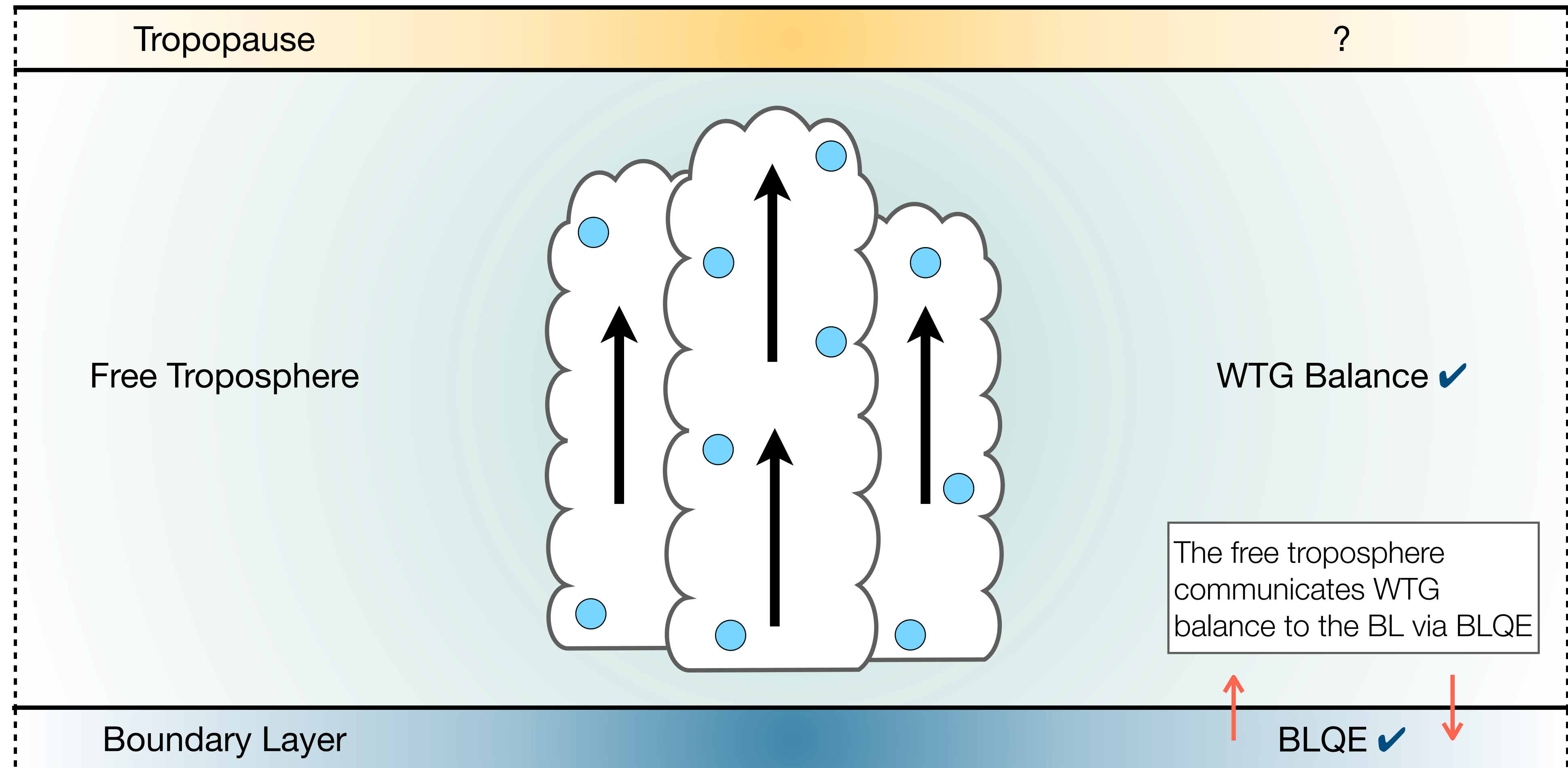


# What now?

WTG balance is only valid in the free troposphere only. Why? (wheel of fortune)



# What now?





# Moist thermodynamics under WTG balance

The previous slides reveal that regardless of view, the strict WTG circulation is related to moisture and temperature. Let's examine the MSE budget

$$\frac{\partial}{\partial t} \left( C_p T + L_v q \right) \simeq - \nabla \cdot \mathbf{u} \text{MSE} + Q_r - \frac{\partial F_{\text{MSE}}}{\partial p}$$

Which we can column integrate to obtain the following:

$$\frac{\partial}{\partial t} \left( C_p \langle T \rangle + L_v \langle q \rangle \right) = - \nabla_h \cdot \langle \mathbf{v} \text{MSE} \rangle + \langle Q_r \rangle + L_v E + \text{SHF}$$

Side note: Did you know that Larissa Back was the first to use the MSE budget in large-scale dynamics. I found out last week.

# Nondimensional MSE budget

In non dimensional form we can examine what terms dominate the MSE budget.

$$\hat{\alpha} \frac{\partial \hat{MSE}}{\partial \hat{t}} = - \hat{\alpha} \frac{Ro}{Ro_\tau} \hat{\mathbf{v}} \cdot \hat{\nabla} \hat{MSE} - \hat{\omega} \frac{\partial \hat{MSE}}{\partial \hat{p}} + \hat{Q}_r - \hat{\alpha} \frac{\partial \hat{F}_m}{\partial \hat{p}}$$

$$\hat{MSE} \sim N_{mode} \hat{DSE} + \hat{q} \quad N_{mode} \equiv \frac{N_w}{\hat{\alpha}(1 - \hat{\alpha})} \quad \hat{\alpha} = - \frac{\partial_p L_v q}{\partial_p DSE} \sim RH$$

There are no dominant terms in the budget, but what dominates the MSE tendency itself is represented by a non dimensional  $N_{mode}$  scale.



# Nondimensional MSE budget

$$N_{mode} \equiv \frac{N_w}{\hat{\alpha}(1 - \hat{\alpha})}$$

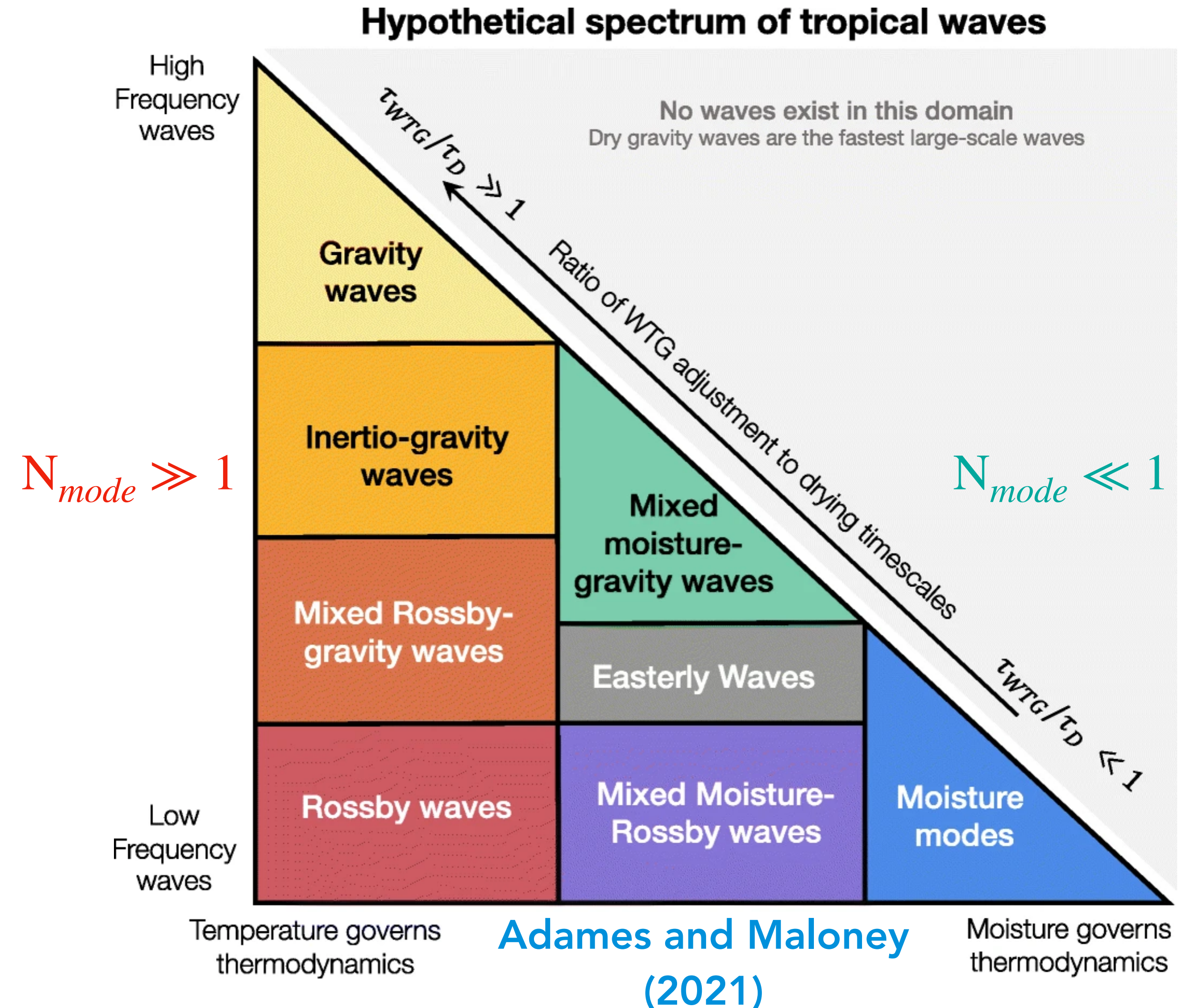
In humid regions of the tropics  $1 - \hat{\alpha} \sim 0.1$ , so that we can be in WTG balance and temperature (DSE) is non-negligible. It's still important to the thermodynamics.

Moisture becomes dominant when  $N_w \sim 0.01$ .

# Under WTG balance: Tropical weather systems are diverse

Moist thermodynamics are determined by:

$$N_{mode} = \frac{DSE'}{L_v q'} \sim 10 \frac{\tau_g^2}{\tau^2}$$

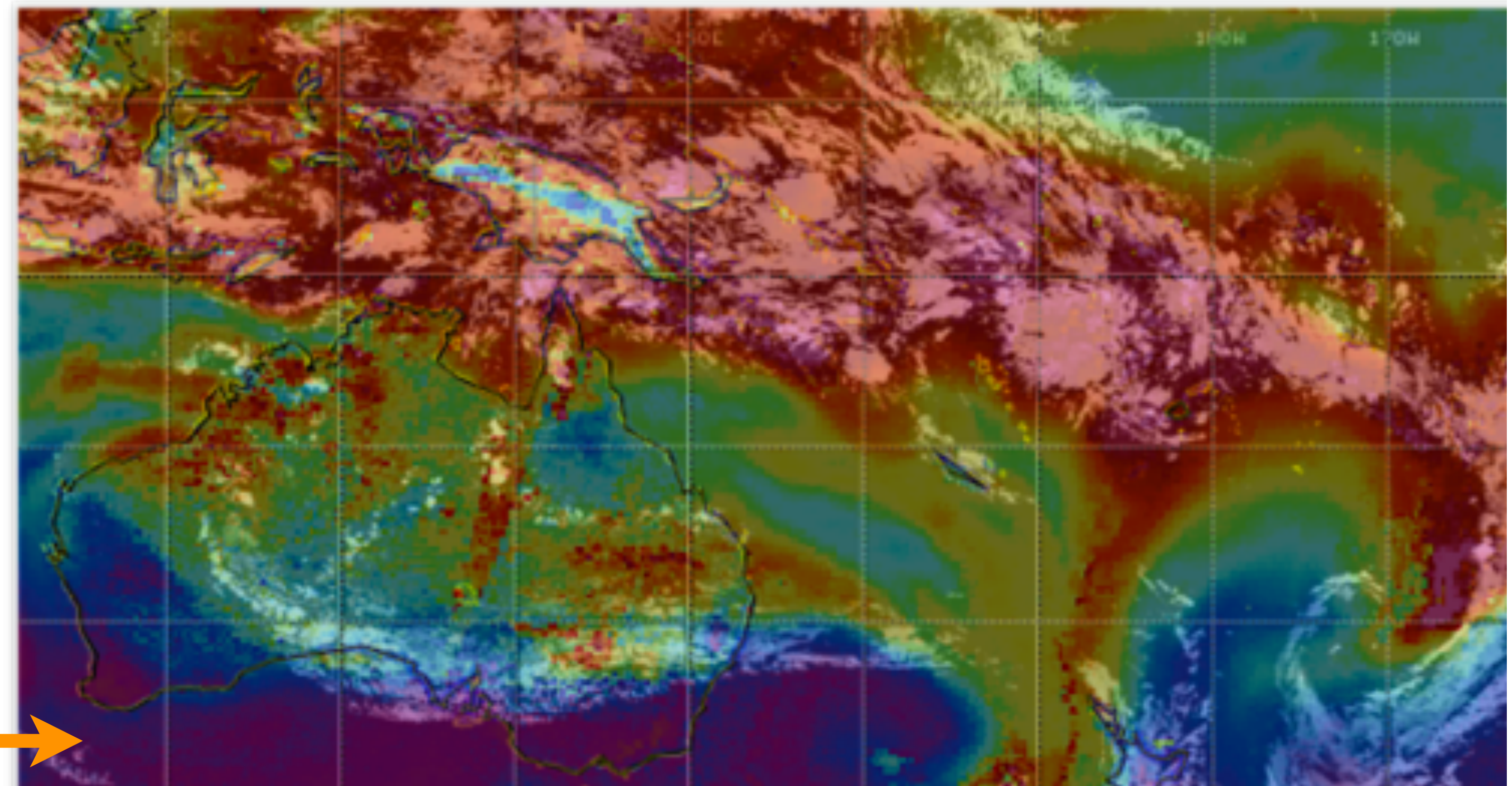
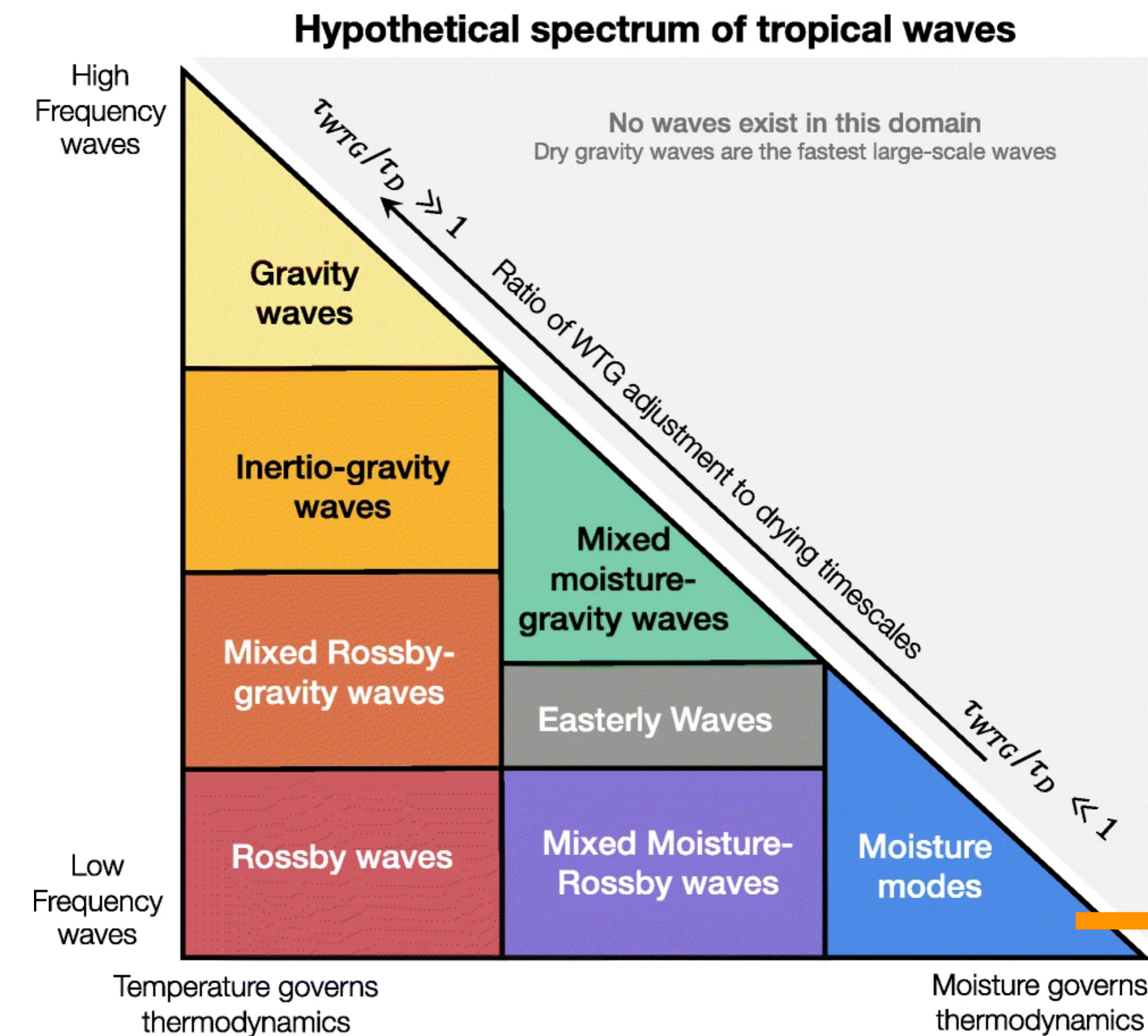




# Moisture Modes

The systems on the left of the diagram are familiar, but the ones on the right are not.

In the motions systems on the right water vapor plays governs their thermodynamics and they are often referred to as moisture modes.

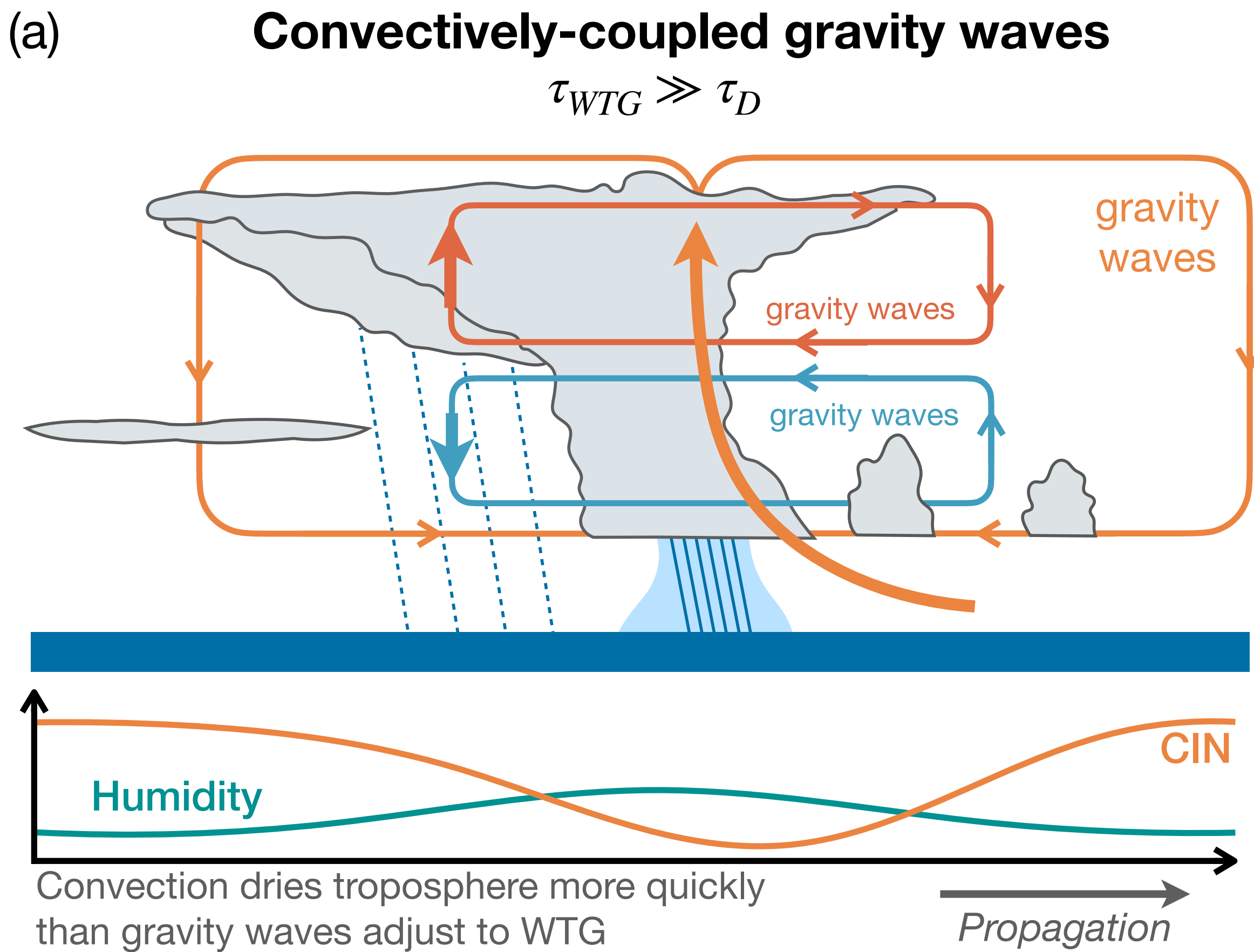




# How do moisture modes arise?

Consider the adjustment to WTG balance

$$N_{mode} \gg 1$$



$$N_{mode} \ll 1$$

