AOS 801: Advanced Tropical Meteorology

Lecture 15   Spring 2023

Tropical Dynamics under WTG balance 3: Consequences

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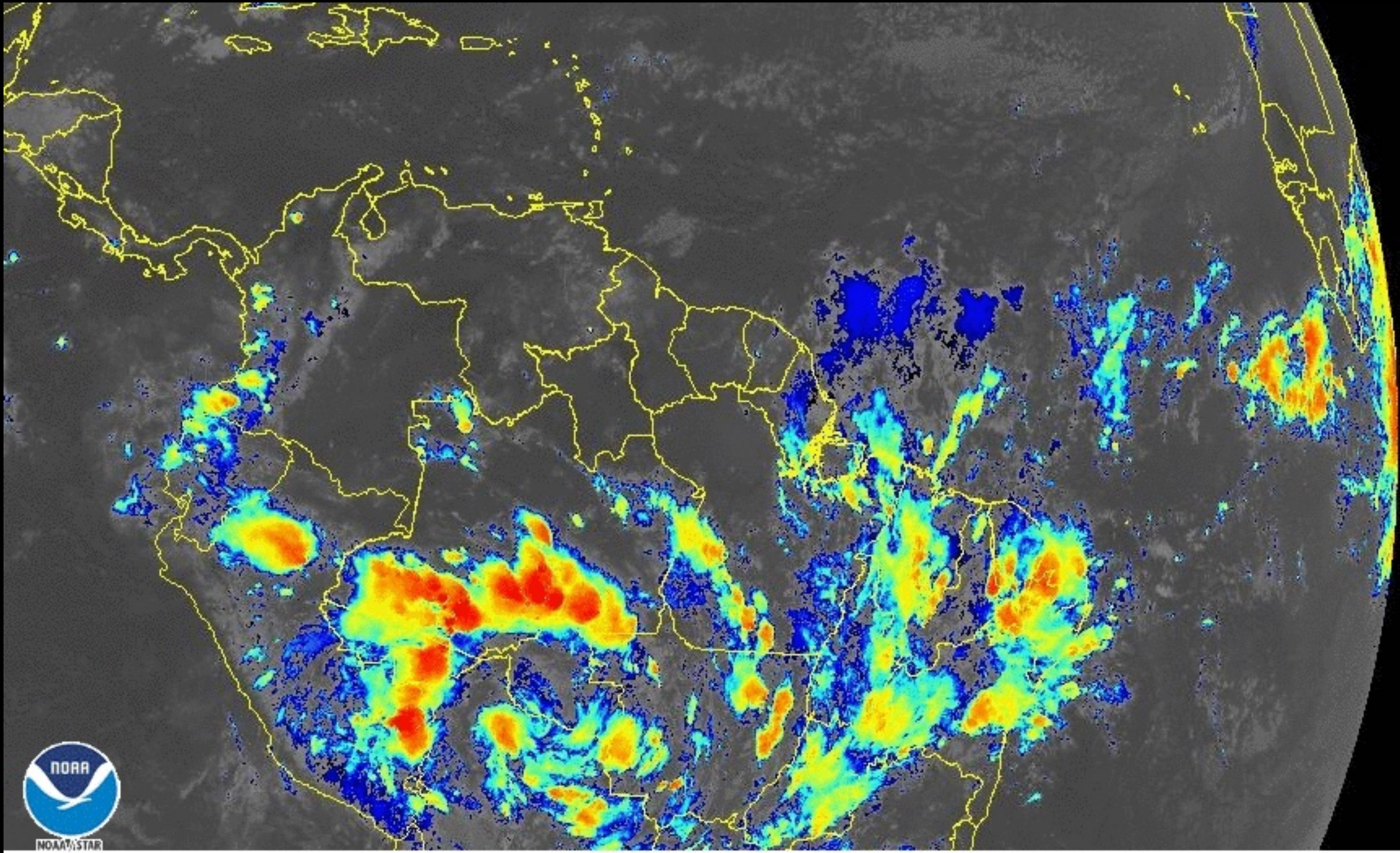
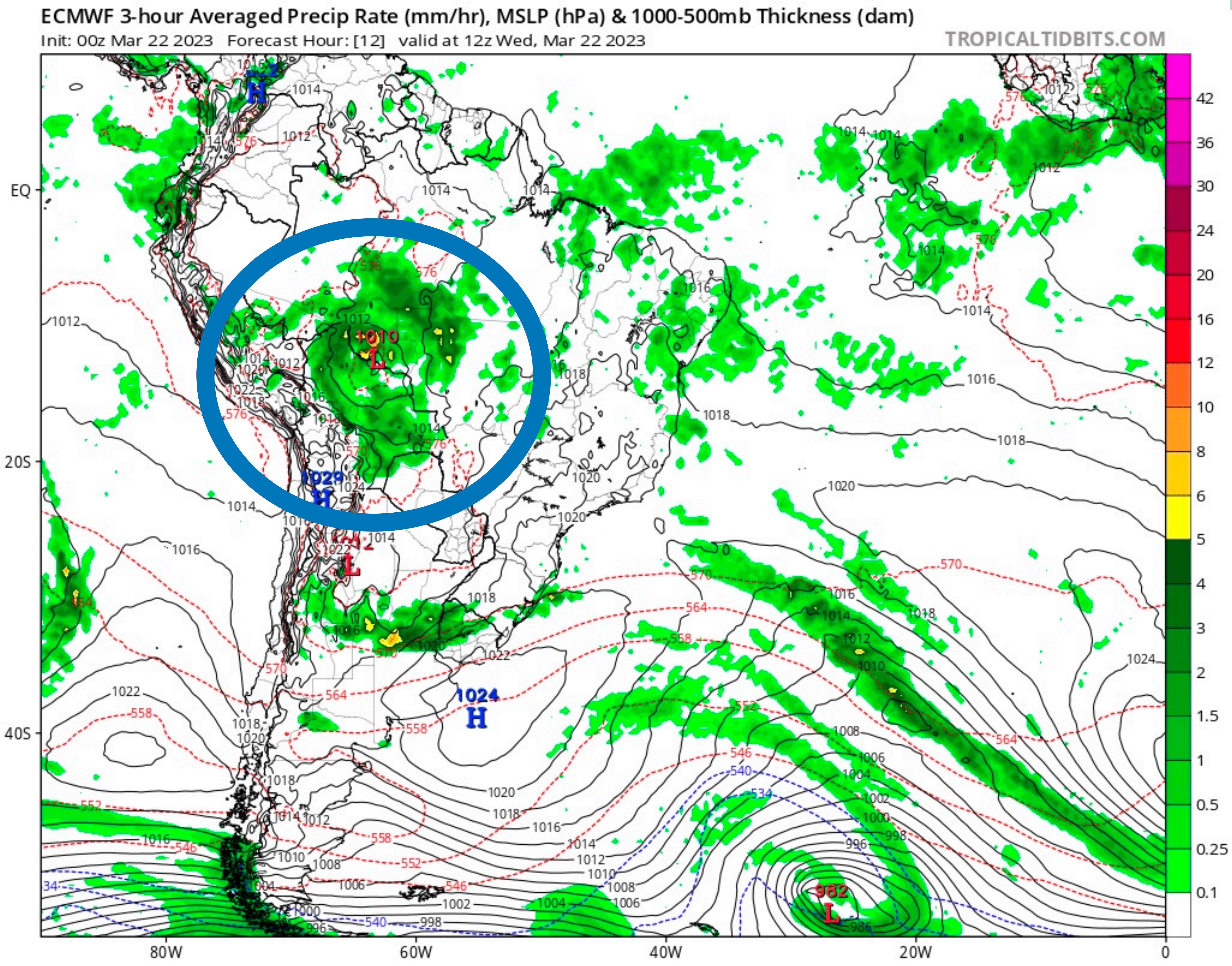
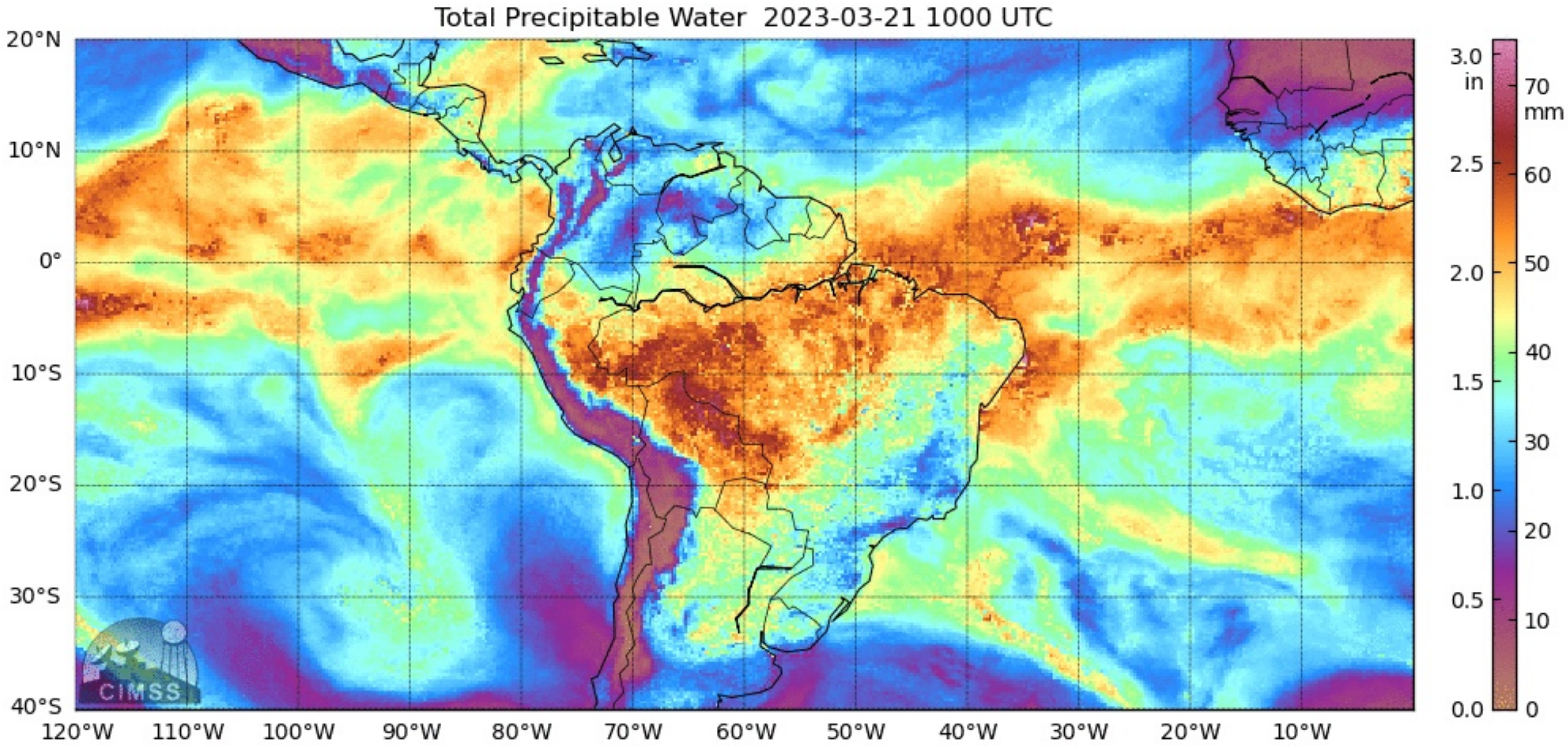


22 Mar 2023 00:20Z - NOAA/NESDIS/STAR - GOES-East - Sandwich Composite

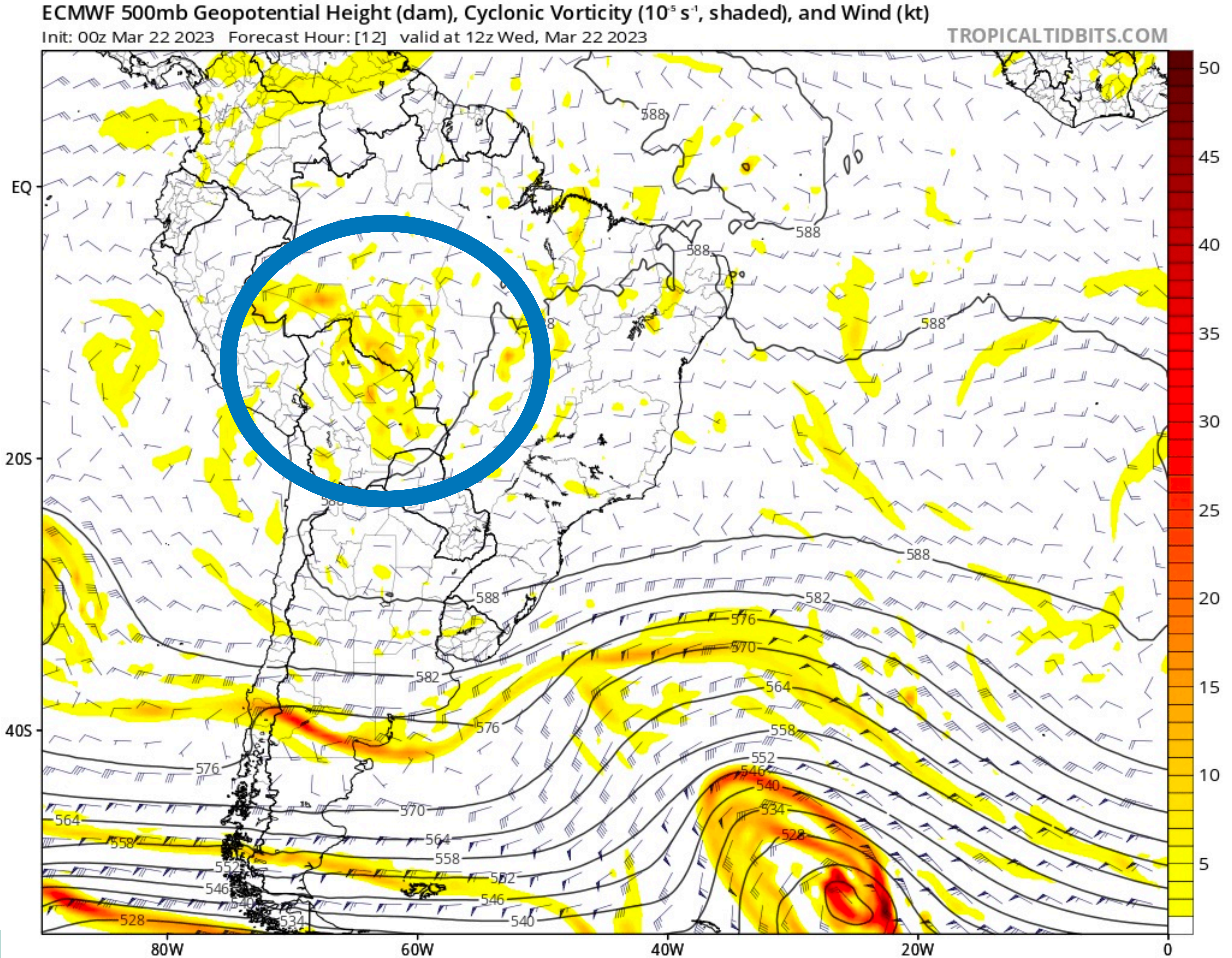
Announcements

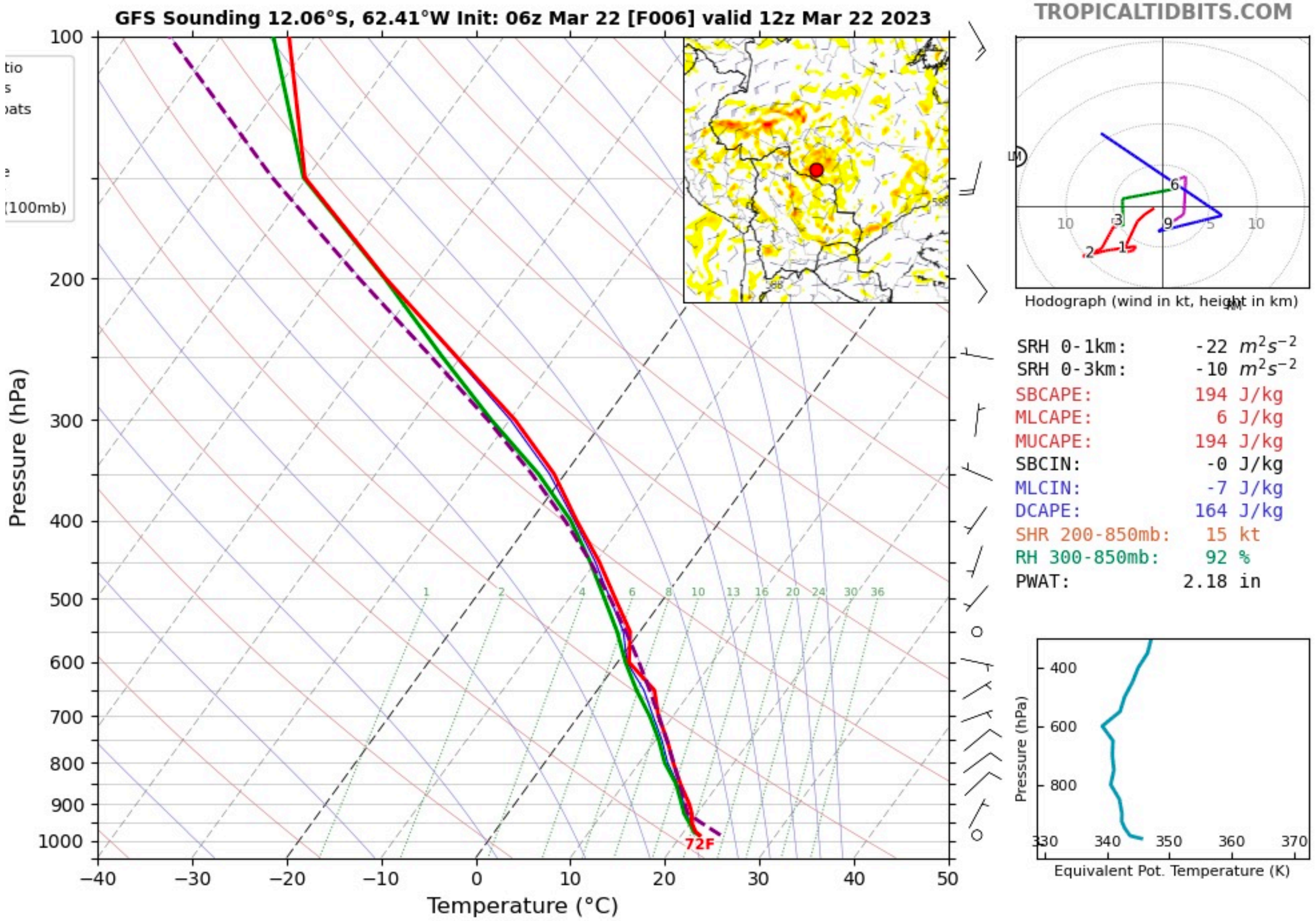


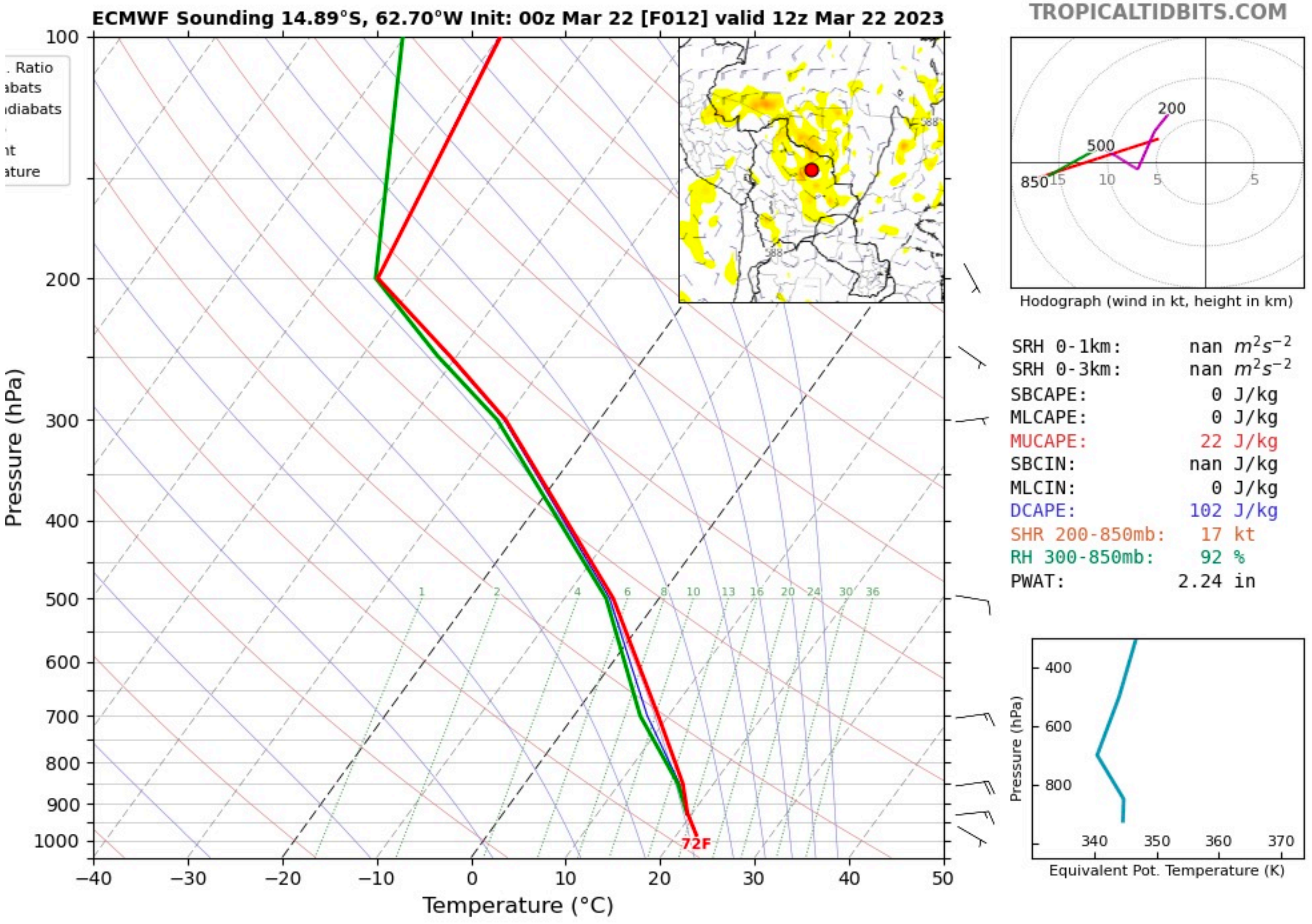
Announcements



http://tropic.ssec.wisc.edu/real-time/mtpw2/product.php?color_type=tpw_nrl_colors&prod=samer1&timespan=120hrs&anim=html5


ECMWF 500mb Geopotential Height (dam), Cyclonic Vorticity (10⁻⁵ s⁻¹, shaded), and Wind (kt)
Init: 00z Mar 22 2023 Forecast Hour: [12] valid at 12z Wed, Mar 22 2023
TROPICALTIDBITS.COM
EQ
20S
40S
80W
60W
40W
20W
0
50
45
40
35
30
25
20
15
10
5


GFS Sounding 12.06°S, 62.41°W Init: 06z Mar 22 [F006] valid 12z Mar 22 2023
TROPICALTIDBITS.COM
Pressure (hPa)
Temperature (°C)
72F
Hodograph (wind in kt, height in km)
SRH 0-1km: -22 m^2s^{-2}
SRH 0-3km: -10 m^2s^{-2}
SBCAPE: 194 J/kg
MLCAPE: 6 J/kg
MUCAPE: 194 J/kg
SBCIN: -0 J/kg
MLCIN: -7 J/kg
DCAPE: 164 J/kg
SHR 200-850mb: 15 kt
RH 300-850mb: 92 %
PWAT: 2.18 in
Equivalent Pot. Temperature (K)


ECMWF Sounding 14.89°S, 62.70°W Init: 00z Mar 22 [F012] valid 12z Mar 22 2023
TROPICALTIDBITS.COM
Pressure (hPa)
Temperature (°C)
72F
Hodograph (wind in kt, height in km)
SRH 0-1km: nan m^2s^{-2}
SRH 0-3km: nan m^2s^{-2}
SBCAPE: 0 J/kg
MLCAPE: 0 J/kg
MUCAPE: 22 J/kg
SBCIN: nan J/kg
MLCIN: 0 J/kg
DCAPE: 102 J/kg
SHR 200-850mb: 17 kt
RH 300-850mb: 92 %
PWAT: 2.24 in
Equivalent Pot. Temperature (K)

Announcements

To have more time, let’s move the paper discussion for Wednesday

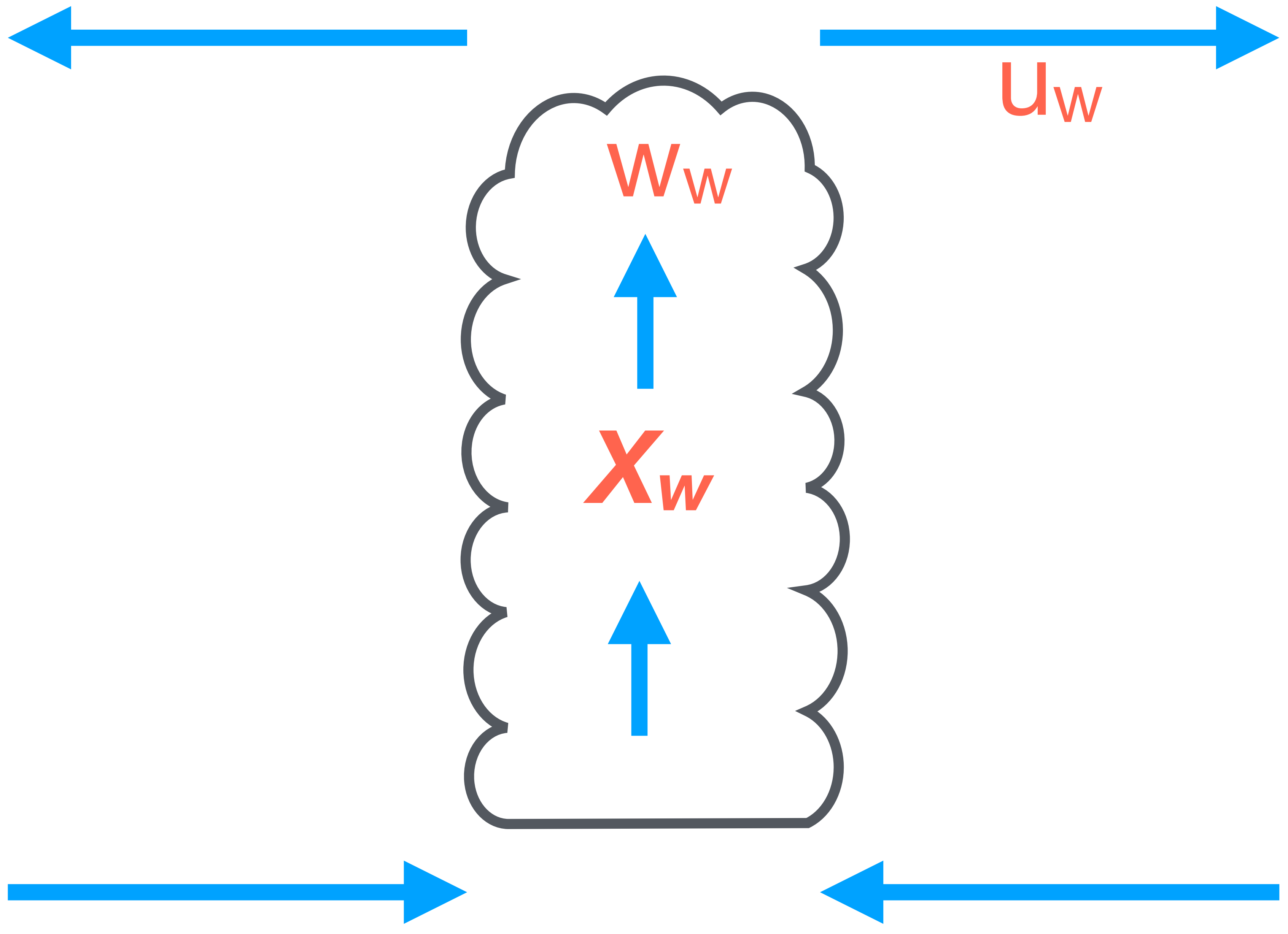
HW3 and PA3 are up, they’re due on April 6.

We found that the irrotational wind is in WTG balance, and the deviation from strict WTG balance is non-divergent:

$$\mathbf{v} = \mathbf{v}_w + \mathbf{v}' \simeq \nabla_h \chi_w + \mathbf{k} \times \nabla_h \psi'$$

$$\mathbf{v}_w = \nabla_h \chi_w$$

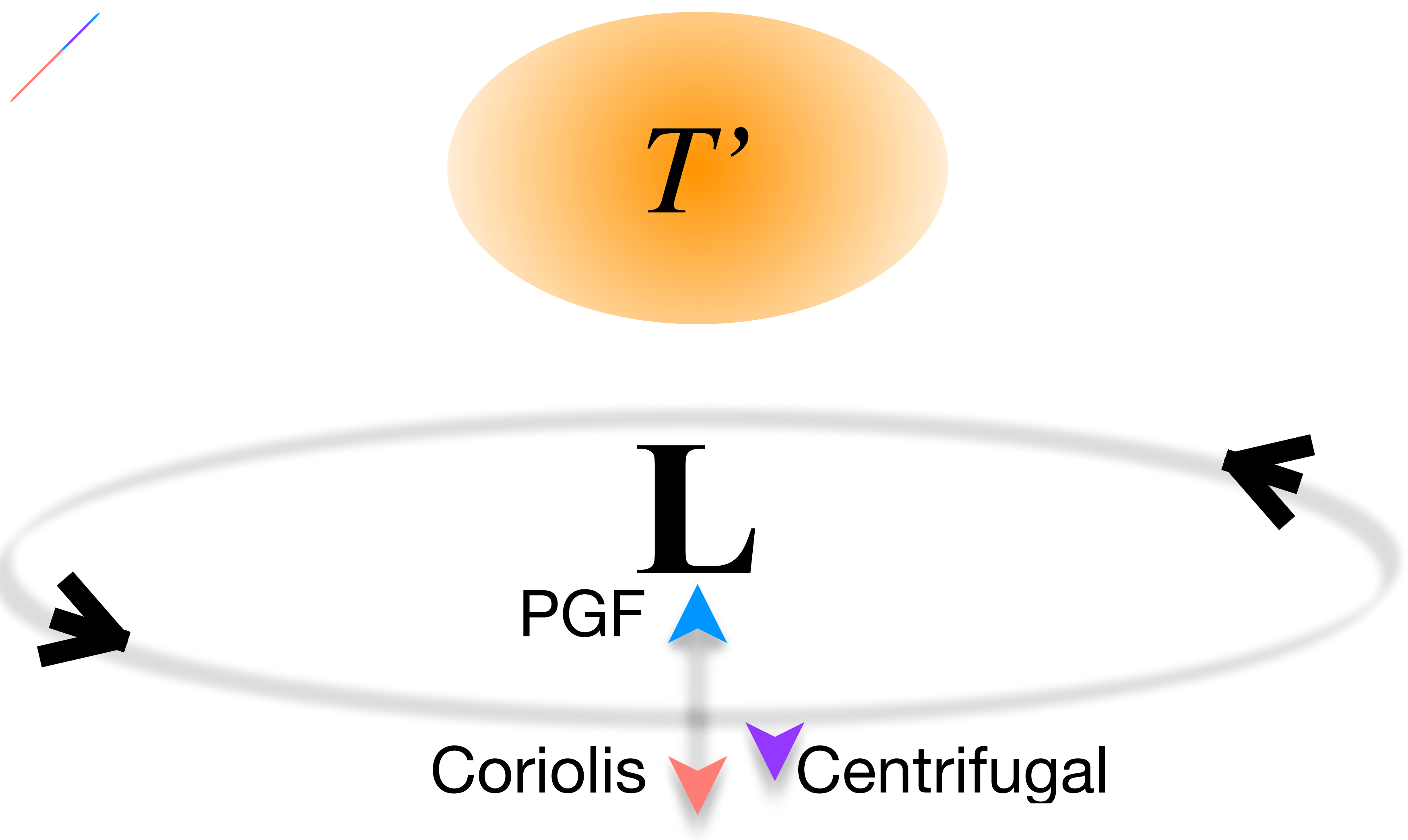
Strict WTG component



$$\mathbf{v}' = \mathbf{k} \times \nabla_h \psi'$$

Deviation from strict WTG balance

Gradient wind, Cyclostrophic, Semi-geostrophic



Basic equations under WTG balance

Under WTG balance, the basic equations reduce to

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left(\boxed{\mathbf{v}\zeta_a} - \boxed{\omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p}} \right)$$

Horizontal absolute vorticity flux

Vertical Flux of Horizontal Vorticity

$$\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\boxed{\mathbf{v}\delta_w} + \boxed{\omega_w \frac{\partial \mathbf{v}}{\partial p}} \right) - \boxed{\Sigma}$$

Horizontal diabatic divergence flux

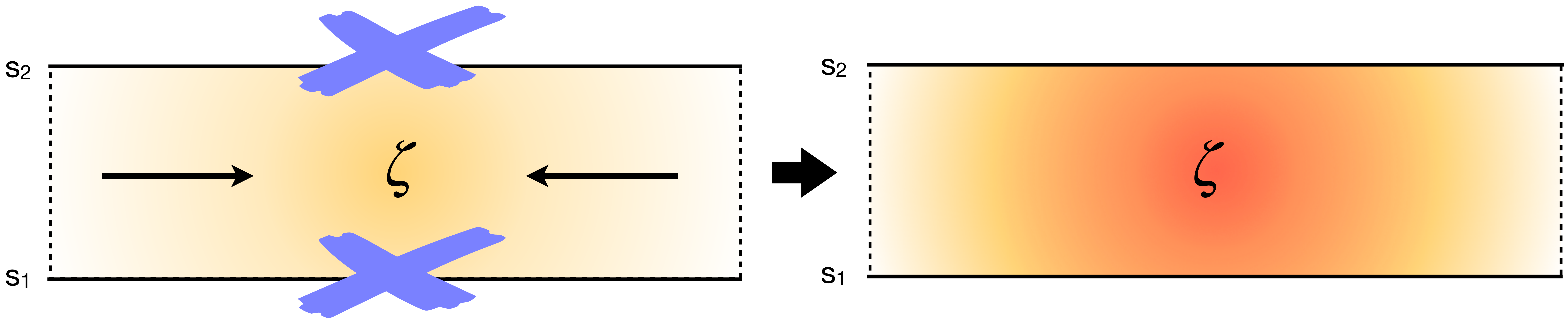
Vertical Momentum Advection

$\boxed{\Sigma} = \nabla_h^2 \Phi + \nabla_h \cdot (f\mathbf{k} \times \mathbf{v}) - 2\mathbf{k} \cdot \left(\nabla_h u \times \nabla_h v \right)$ = Deviation from nonlinear balance

WTG PV equation

WTG balance does **NOT** allow ζ to be fluxed vertically.

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left(\mathbf{v}\zeta_a - \omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p} \right)$$

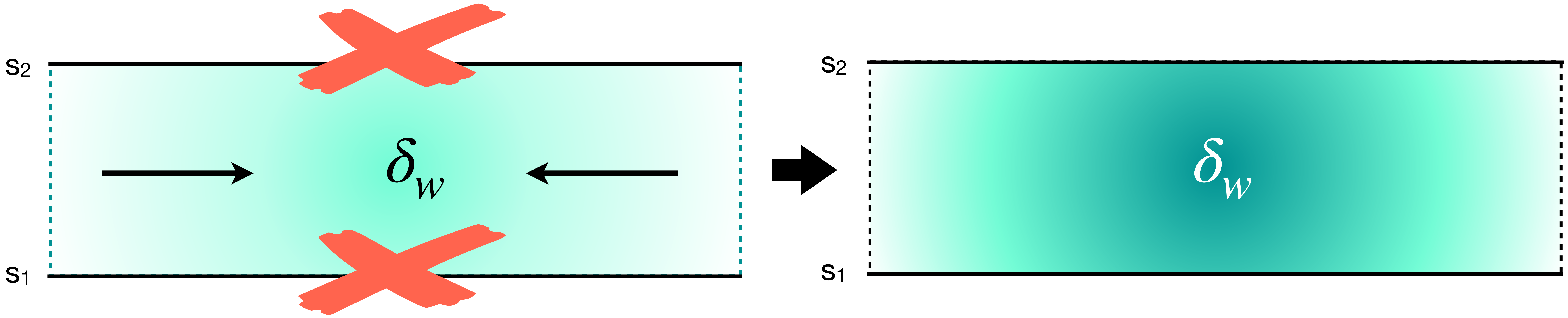


You can only accrete vorticity within single pressure level

WTG divergence equation

WTG balance does **NOT** allow δ_w to be fluxed vertically.

$$\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\nabla_h(\Phi + \mathrm{KE}) - \boldsymbol{\eta}_a \times \mathbf{u} \right)$$



You can only accrete divergence within single pressure level

WTG PV equation

Note that the terms on the rhs of both equations can be interpreted as a horizontal flux

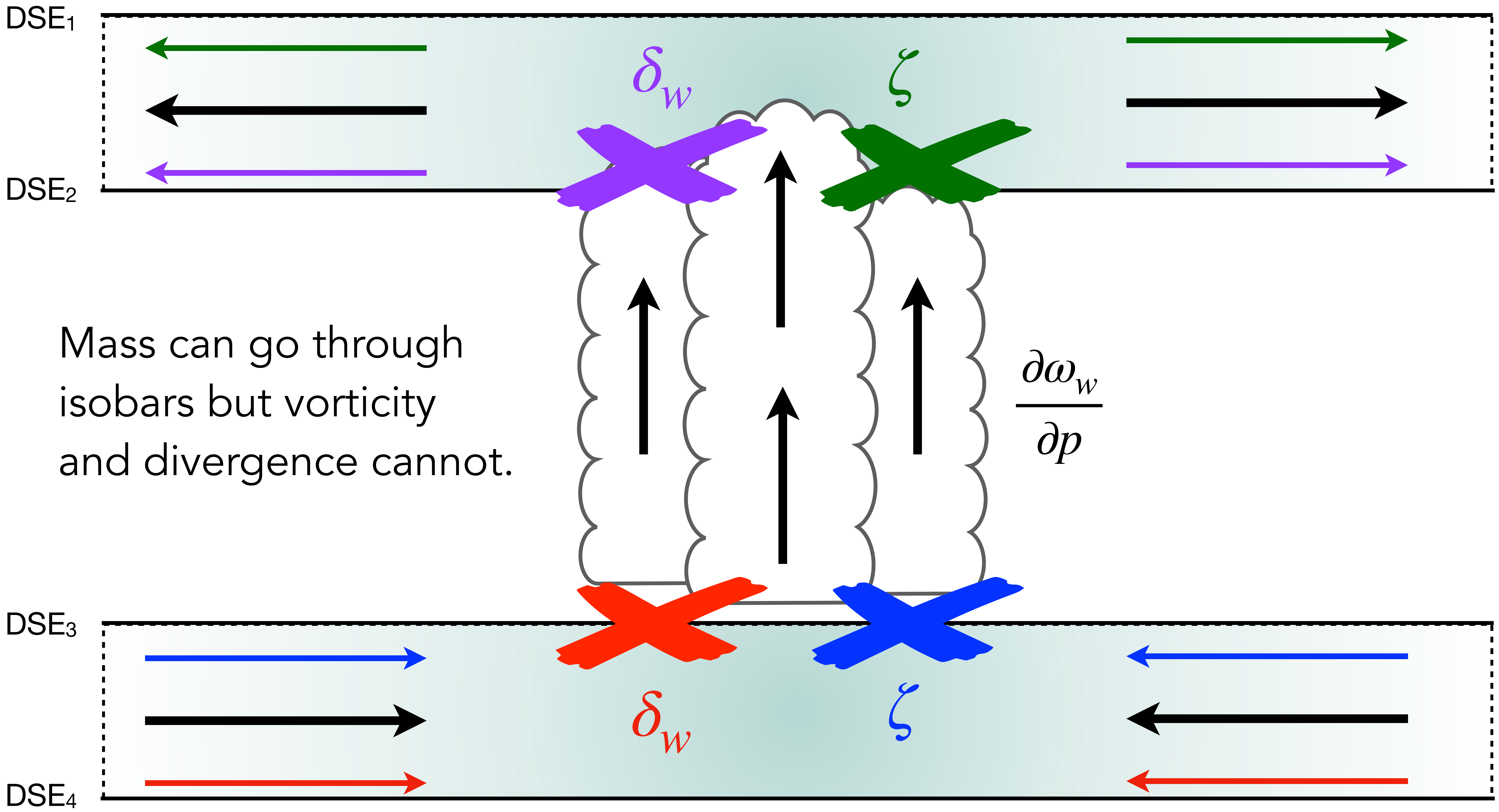
$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \mathbf{Z} \qquad \frac{\partial \delta_w}{\partial t} = \nabla_h \cdot \mathbf{D}$$

Noting that

$$\frac{\partial \omega_w}{\partial p} = -\nabla_h \cdot \mathbf{v}$$

Recall that ω_w is a vertical mass flux

Tropical motions under WTG balance are severely restricted



Mass can go through isobars but vorticity and divergence cannot.

Analogy to TSA

Imagine that parcels in deep convection are taking a flight to the upper troposphere.

The parcels can pass security, but they're not allowed to bring their vorticity and divergence with them.

The vorticity and divergence must stay at the current location (i.e. the level that the parcels leave).

Isobars behave as "strainers"

Mass can go through isobars but vorticity and divergence cannot.

Water: Mass

Rice: Vorticity and Divergence

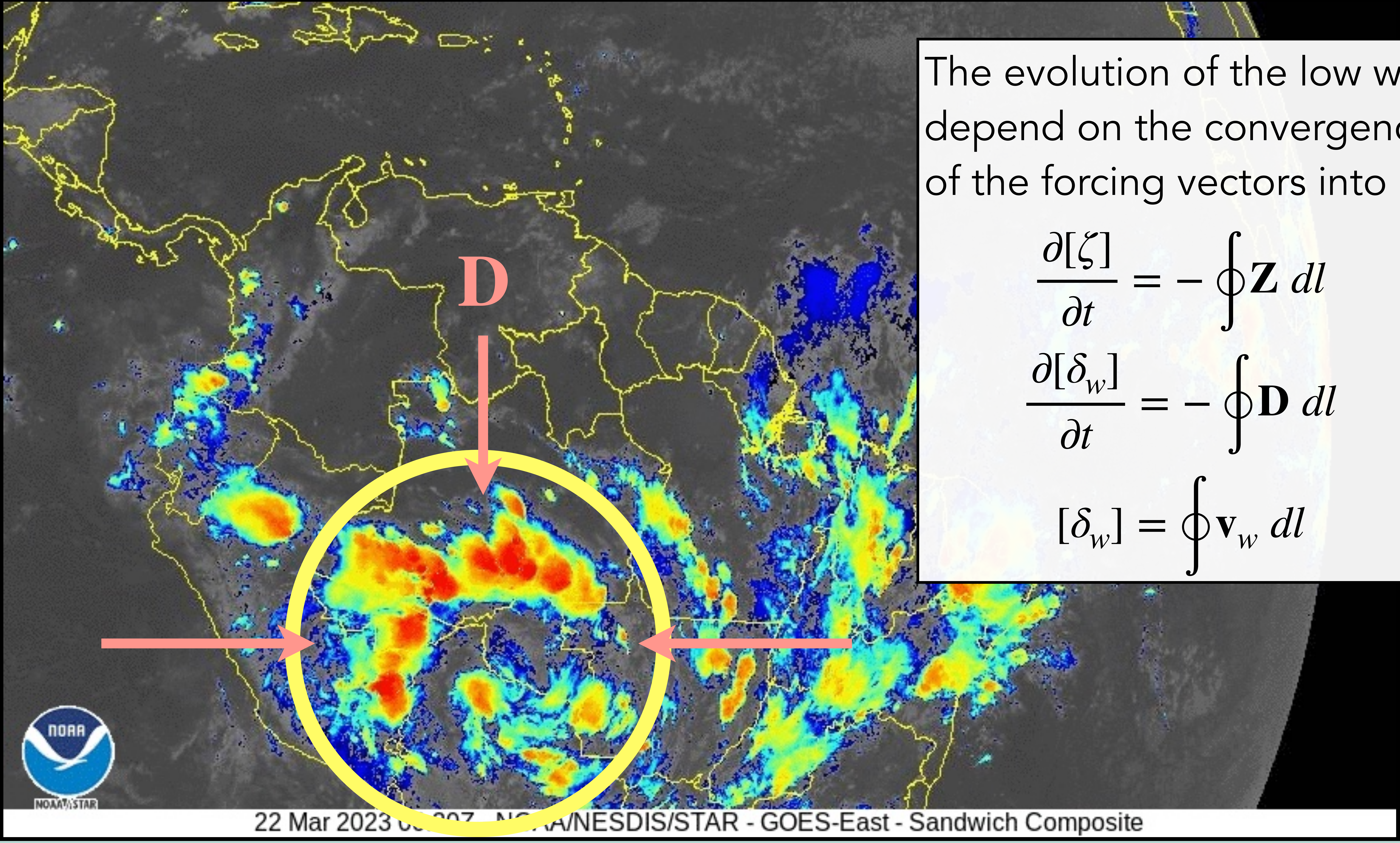
Applying divergence theorem

Let's do an areal integral around a circle of radius r. Using the divergence theorem we find the following

$$\frac{\partial[\zeta]}{\partial t} = \int\int \frac{\partial \zeta}{\partial t} dA \qquad \frac{\partial[\zeta]}{\partial t} = -\int\int \nabla_h \cdot \mathbf{Z}\, dA = -\oint \mathbf{Z}\, dl$$

So we obtain the following relations

$$\frac{\partial[\zeta]}{\partial t} = -\oint \mathbf{Z}\, dl \qquad \frac{\partial[\delta_w]}{\partial t} = -\oint \mathbf{D}\, dl \qquad [\delta_w] = \oint \mathbf{v}_w\, dl$$



The evolution of the low will depend on the convergence of the forcing vectors into it:

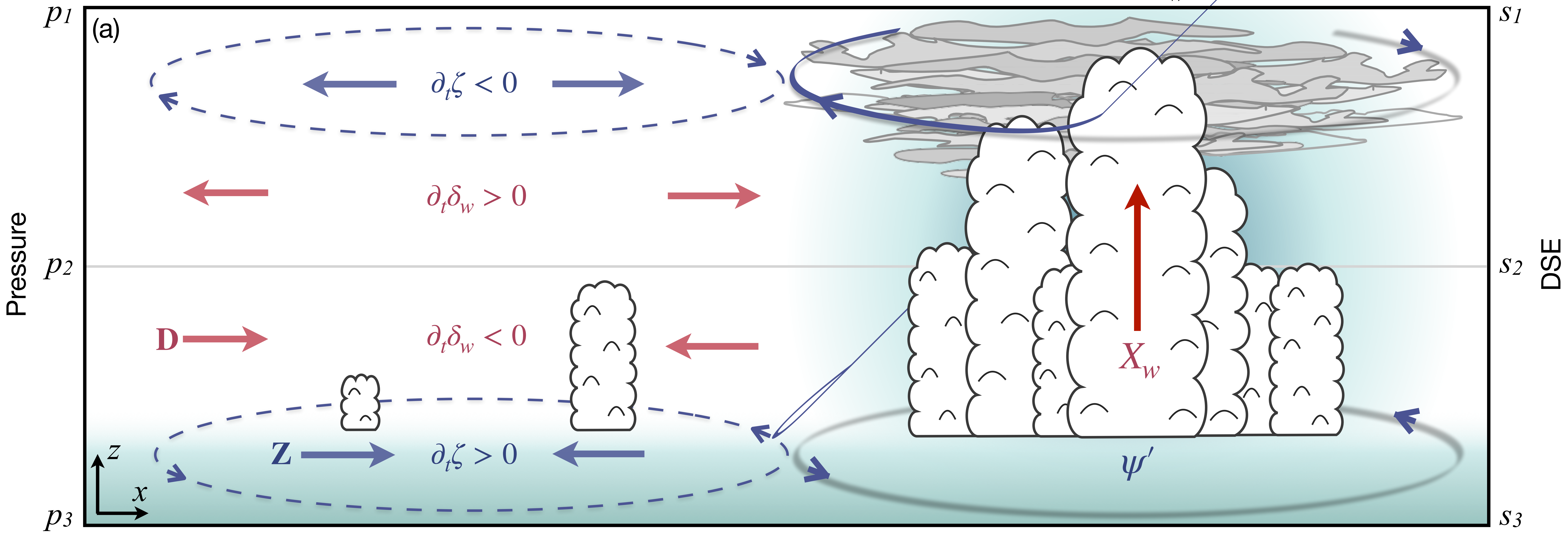
$$\frac{\partial[\zeta]}{\partial t} = -\oint \mathbf{Z}\, dl$$

$$\frac{\partial[\delta_w]}{\partial t} = -\oint \mathbf{D}\, dl$$

$$[\delta_w] = \oint \mathbf{v}_w\, dl$$

Basic equations under WTG balance

Hypothetical evolution of a system that is in WTG and nonlinear balance in X_w and ψ', respectively



The total horizontal flow must evolve in the horizontal plane

Using the divergence theorem again but using the definition of vort and div yields:

$$\frac{\partial[\zeta]}{\partial t} = \int\int \nabla_h \cdot \nabla_h \psi \, dA = \oint \nabla_h \psi \, dl$$

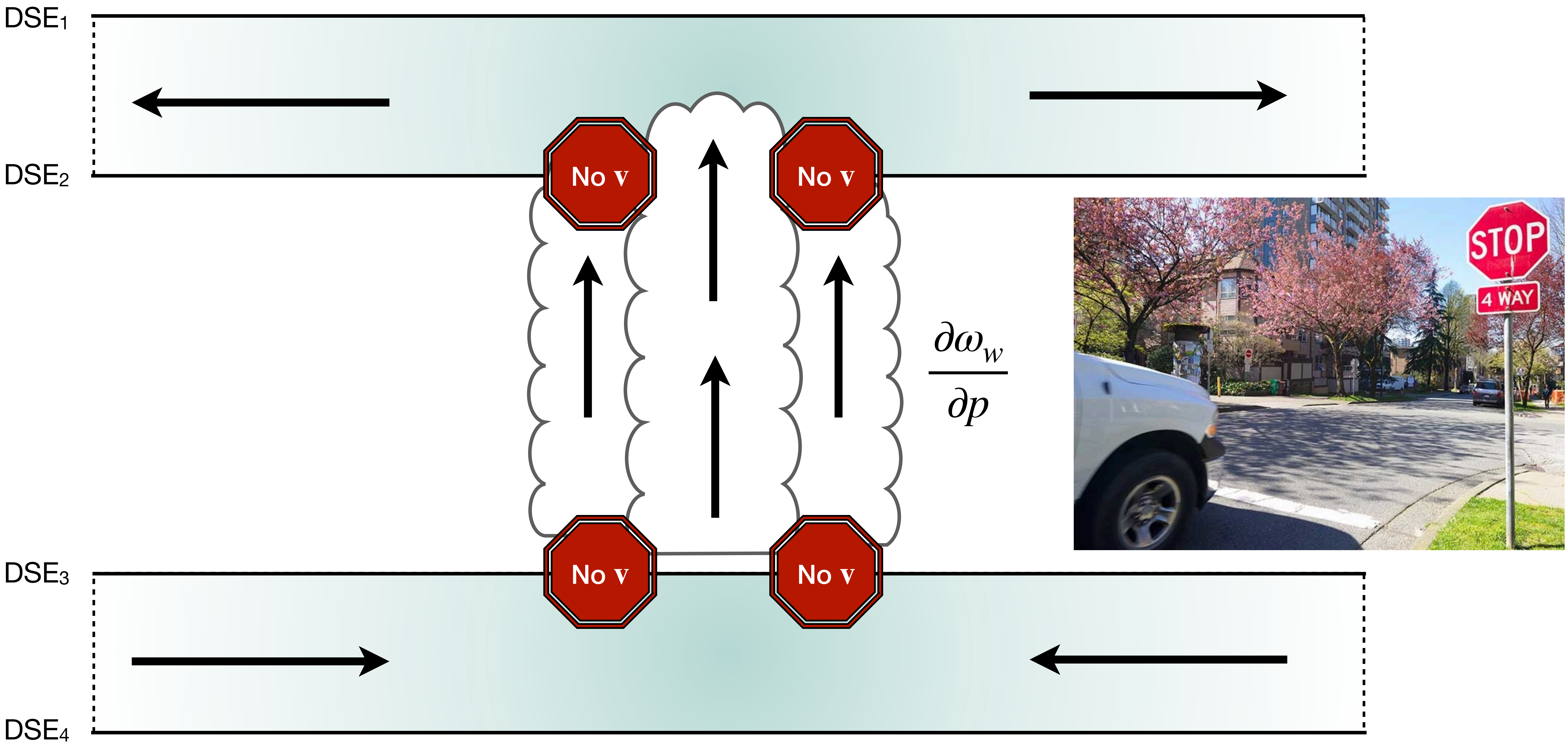
$$\frac{\partial[\delta_w]}{\partial t} = \int\int \nabla_h \cdot \nabla_h \chi_w \, dA = \oint \nabla_h \chi_w \, dl$$

After doing some math we find that

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{D} - \mathbf{k} \times \mathbf{Z}$$

Horizontal momentum is also restricted to evolve in the horizontal plane.

The horizontal circulation is restricted to evolve at each isobar only



$$\frac{\partial \omega_w}{\partial p}$$

Wrapping it up

While the motion is restricted to evolve in the horizontal plane, the isobars are still coupled:

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left(\mathbf{v}\zeta_a - \boxed{\omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p}} \right)$$

$$\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\mathbf{v}\delta_w + \boxed{\omega_w \frac{\partial \mathbf{v}}{\partial p}} \right) - \Sigma$$

The three dimensional structure then tells you how

Dry motions

In the absence of diabatic heating, the equations reduce to a barotropic vorticity equation

$$\frac{\partial \zeta}{\partial t} = -\mathbf{v} \cdot \nabla_h \zeta_a$$

The isobars behave like barotropic layers. They are decoupled from each other.

This result was found by Charney (1963).

NOTES AND CORRESPONDENCE

NOTES AND CORRESPONDENCE

A Note on Large-Scale Motions in the Tropics[1]

Jule G. Charney

Massachusetts Institute of Technology

16 August 1963