Motion under WTG balance

WTG vorticity

$$\boxed{\frac{\partial \zeta_a}{\partial t} = -\nabla_h \cdot \left(\vec{v}\zeta_a - \hat{k}\,\omega_w \times \frac{\partial \vec{v}}{\partial p} \right)}$$

$$\zeta_a = \zeta + f$$

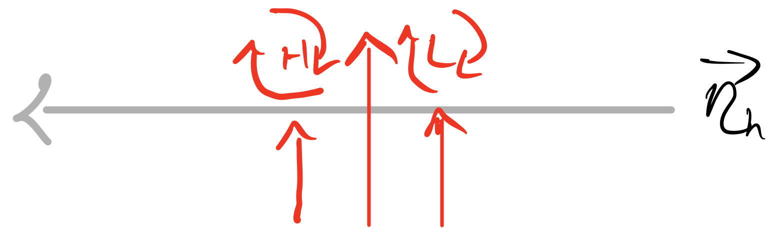
hor. flux of absolute vort.

vertical flux of hor vorticity

"tilting effect"

$$\vec{\eta}_h = \hat{k} \times \frac{\partial \vec{v}}{\partial p} = \frac{\partial v}{\partial p}\hat{i} - \frac{\partial u}{\partial p}\hat{j}$$

$\omega_w \vec{\eta}_h$



$$\boxed{\frac{\partial \delta_w}{\partial t} = -\nabla_h \cdot \left(\vec{v}\delta_w + \omega_w \frac{\partial \vec{v}}{\partial p} \right) - \Sigma}$$

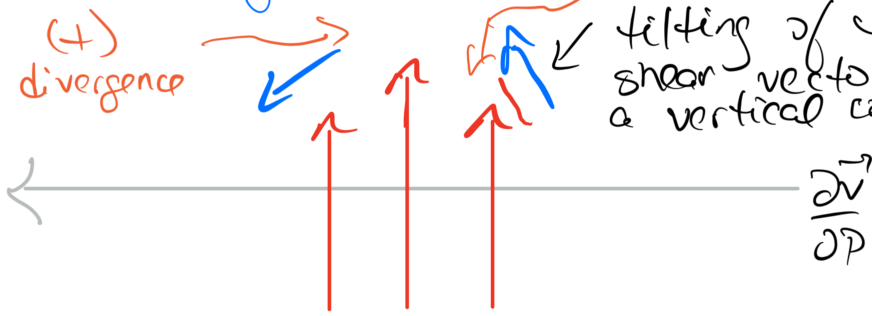
hor. flux of WTG divergence

vertical mom. adv.

convergence

(+) divergence

tilting of the shear vector creates a vertical component



Imagine a parcel with mass M
But has the following properties
ζ, δ, $\vec{v}$, q, etc.

Now ζ and δ are properties of a parcel.

* Imagine that the parcel has an attire δ on ζ like a water bottle and shampoo.

Let's write the δ and ζ_a eqns. in the form of forcing vectors

(1) $\dfrac{\partial \zeta_a}{\partial t} = -\nabla_h \cdot \vec{Z}$

(2) $\dfrac{\partial \delta}{\partial t} = \nabla_h \cdot \vec{D}$

$$\vec{Z} = \vec{V}\zeta_a - \omega \hat{k} \times \frac{\partial \vec{V}}{\partial p}$$

$$\vec{D} = -\nabla_h(\Phi + KE) - \vec{\eta}_a \times \vec{V}$$

$$KE = \frac{1}{2}(u^2 + v^2)$$

$$\eta_a = \frac{\partial v}{\partial p}\hat{i} - \frac{\partial u}{\partial p}\hat{j} + \zeta_a \hat{k}$$

Let's integrate over an area A and invoke the div theorem:

$$\iint \frac{\partial \zeta_a}{\partial t} dA = -\iint \nabla_h \cdot \vec{Z} \, dA$$

Divergence theorem

$$= -\oint \vec{Z} \, dl$$

$$\iint \frac{\partial \delta_w}{\partial t} dA = \oint \vec{D} \, dl$$

$$\frac{\partial [\zeta_a]}{\partial t} = -\oint \vec{Z} \, dl \qquad \frac{\partial [\delta]}{\partial t} = \oint \vec{D} \, dl$$