# AOS 801: Advanced Tropical Meteorology

*Lecture 14   Spring 2023*

## Tropical Dynamics under WTG balance 2: Consequences

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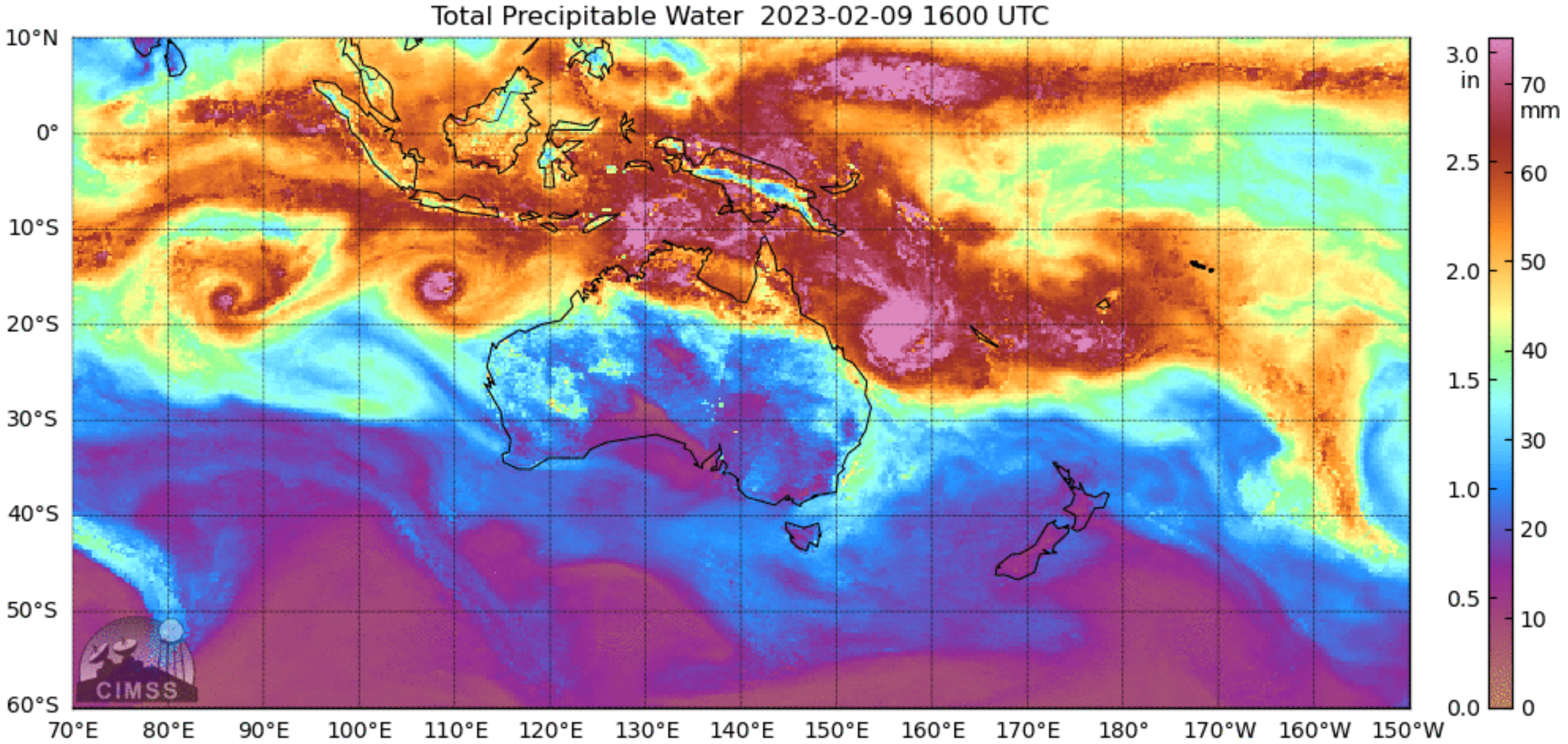
# Announcements

HW3 and PA3 are up, they're due in two weeks.

Paper discussion next Monday.

# Remember?



Total Precipitable Water 2023-02-09 1600 UTC
10°N
0°
10°S
20°S
30°S
40°S
50°S
60°S
70°E
80°E
90°E
100°E
110°E
120°E
130°E
140°E
150°E
160°E
170°E
180°
170°W
160°W
150°W
3.0 in
2.5
2.0
1.5
1.0
0.5
0.0
70 mm
60
50
40
30
20
10
0
CIMSS

# Post spring break recap

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$

Given that there's an adjustment process associated with gravity waves, we must assume systems that are in WTG balance must evolve slowly.

But **how slowly**? We must invoke scale analysis to find out!

# WTG adjustment timescale

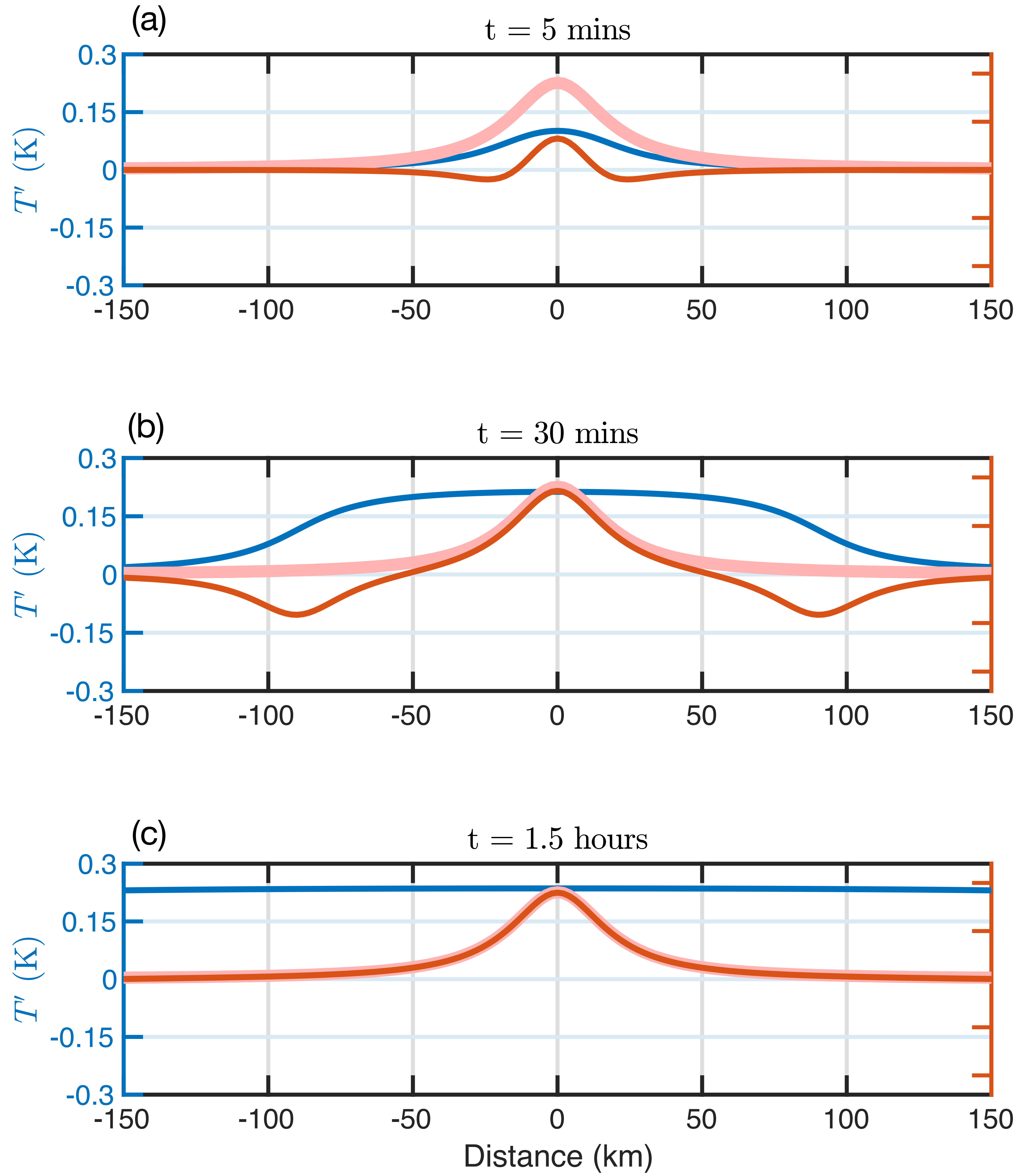
In the case of fixed heating, WTG is attained when the gravity waves exit the region of interest.

This is known as the **gravity wave adjustment timescale**

$$\tau_g = \frac{L}{c}$$

$L$ = domain size

$c$ = gravity wave phase speed




Adames and Maloney (2021)

# Conditions for WTG balance

From examination of the non-dimensional thermodynamic equation we see that $\mathrm{N}_w \ll 1$ for WTG balance to be valid:

$$\mathrm{N}_w = \max\left(\frac{c_p^2}{c^2}, \frac{L_y^2}{L_d^2}\right) \ll 1$$

Condition 1: The system must evolve much more slowly than free gravity waves.

Condition 2: The meridional scale of the system must be smaller than the Rossby radius of deformation.

What do these mean physically?

# Conditions for WTG balance

We can use the definition of gravity wave adjustment timescale

$$N_w = \tau_g^2 \max\left(\tau^{-2}, f^2\right) \ll 1$$

Condition 1: The system must evolve much more slowly than free gravity waves.

Condition 2: The meridional scale of the system must be smaller than the Rossby radius of deformation.

What do these mean physically?

# Systems in WTG balance

In PA3, you will see that in systems that obey WTG balance:

$$\mathrm{Ro}_\tau = \frac{1}{f\tau} \sim 1$$

So that the terms in parenthesis in Nw are similar, so either can be used to understand the motions:

$$\mathrm{N}_w = \tau_g^2 \tau^{-2} \ll 1$$

There is one exception which will be discussed in future lectures

# Basic equations

In order to understand when and why WTG is valid, we must scale the entire set of basic equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla_h)\mathbf{v} + \omega \frac{\partial \mathbf{v}}{\partial p} = -f\mathbf{k} \times \mathbf{v} - \nabla_h \Phi$$

Horizontal Momentum

$$\frac{\partial \Phi}{\partial p} = -\frac{R_d T}{p}$$

Hydrostatic

$$\frac{\partial \omega}{\partial p} = -\nabla_h \cdot \mathbf{v}$$

Mass Continuity

$$\frac{\partial C_p T}{\partial t} + \mathbf{v} \cdot \nabla_h C_p T + \omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$

Thermodynamic

$$\frac{\partial L_v q}{\partial t} + \mathbf{v} \cdot \nabla_h L_v q + \omega \frac{\partial L_v q}{\partial p} = -Q_2$$

Moisture

# The WTG circulation

Let us decompose the wind field into a strict WTG component and a deviation from strict WTG balance.

$$\mathbf{u} = \boxed{\mathbf{u}_w} + \boxed{\mathbf{u}'}$$

Strict WTG Deviation

$$\mathbf{u} = u\mathbf{i} + v\mathbf{j} + \omega\mathbf{k}$$

$$\mathbf{u}_w \cdot \nabla \mathrm{DSE} \equiv Q_1$$

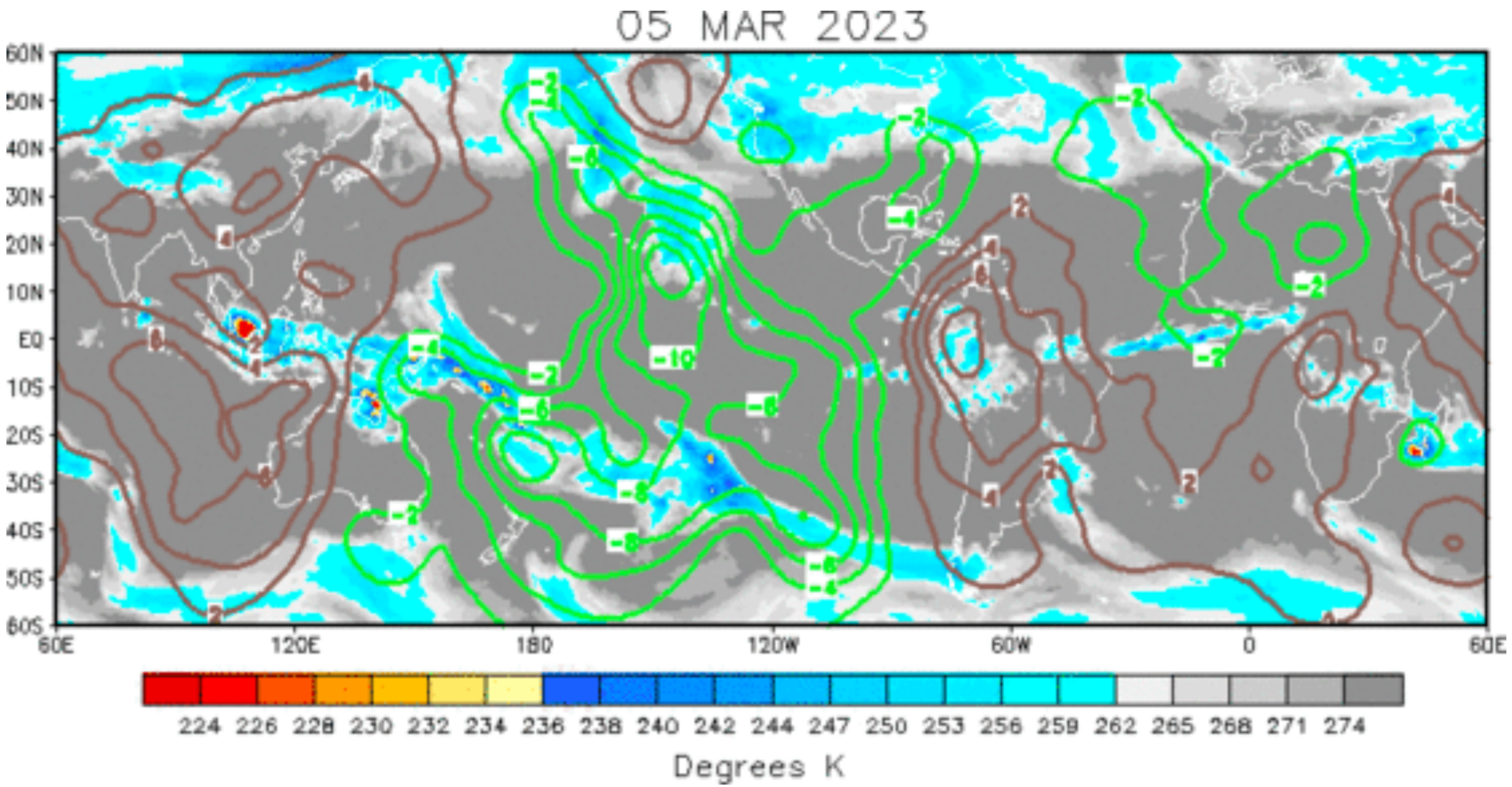
This wind is not associated with any temperature anomalies. Thus it must be purely irrotational:

$$\frac{\partial \omega_w}{\partial p} = -\delta_w = -\nabla_h^2 \chi_w .$$

$$\mathbf{v}_w = \nabla_h \chi_w$$

$\chi_w$ is the **WTG velocity potential**

# Velocity potential



# The integrated velocity potential

Let us define the integrated velocity potential:

$$X_w(x, y, p, t) \equiv \int_0^p \chi_w(x, y, p', t)\mathrm{d}p' \,.$$

A definition that we can use to redefine the WTG circulation as:

$$\left(\frac{\partial \mathrm{DSE}}{\partial x}\frac{\partial^2}{\partial x \partial p} + \frac{\partial \mathrm{DSE}}{\partial y}\frac{\partial^2}{\partial y \partial p} - S_p \nabla_h^2\right) X_w = -\, Q_1 \qquad S_p = -\frac{\partial \mathrm{DSE}}{\partial p}$$

If the horizontal DSE gradient is weak we obtain the following

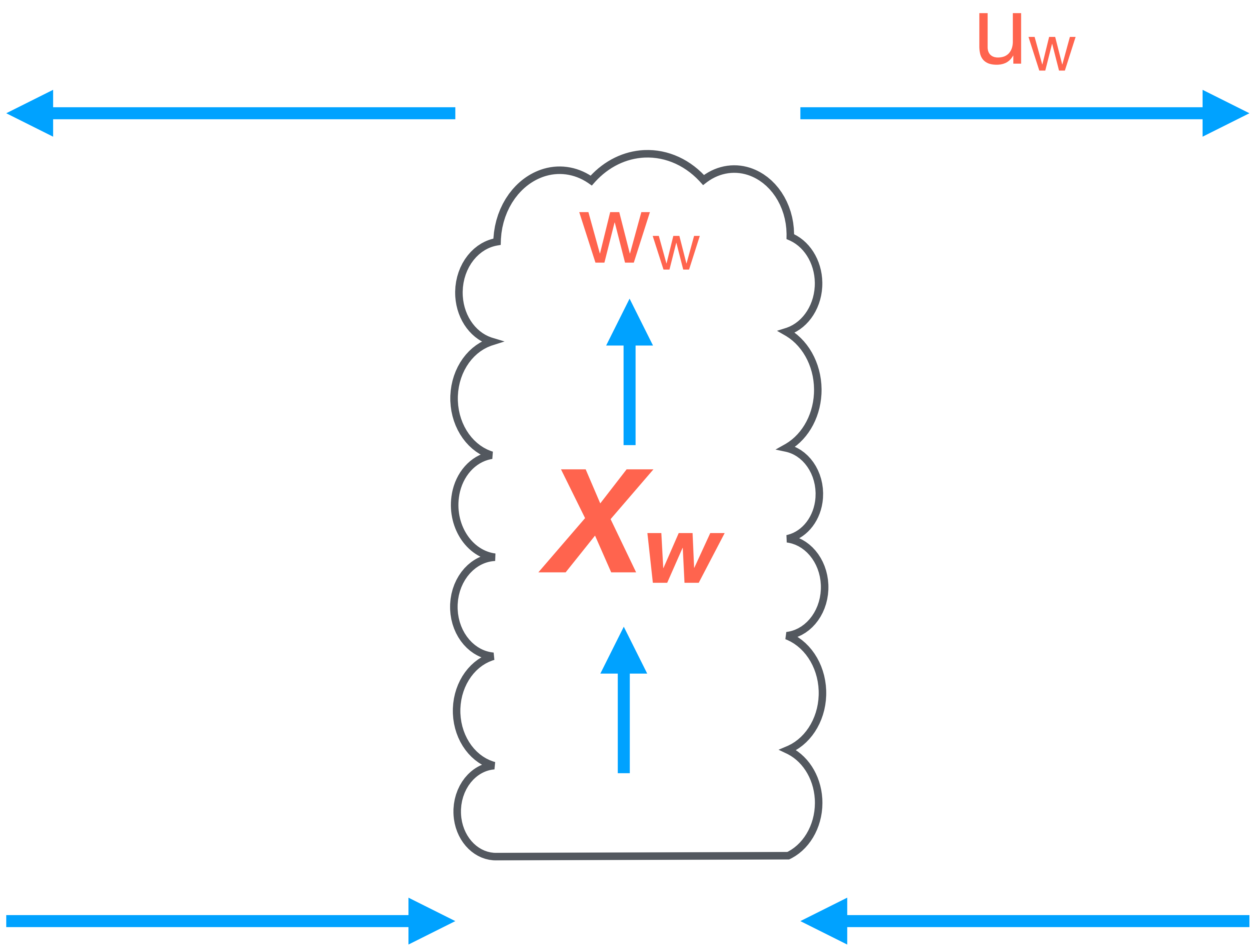
$$S_p \nabla_h^2 X_w = Q_1$$

# The strict WTG circulation

Why bother to define WTG balance this way?

$$S_p \nabla_h^2 X_w = Q_1$$

Because all components of the WTG wind field can be understood in terms of the WTG circulation.



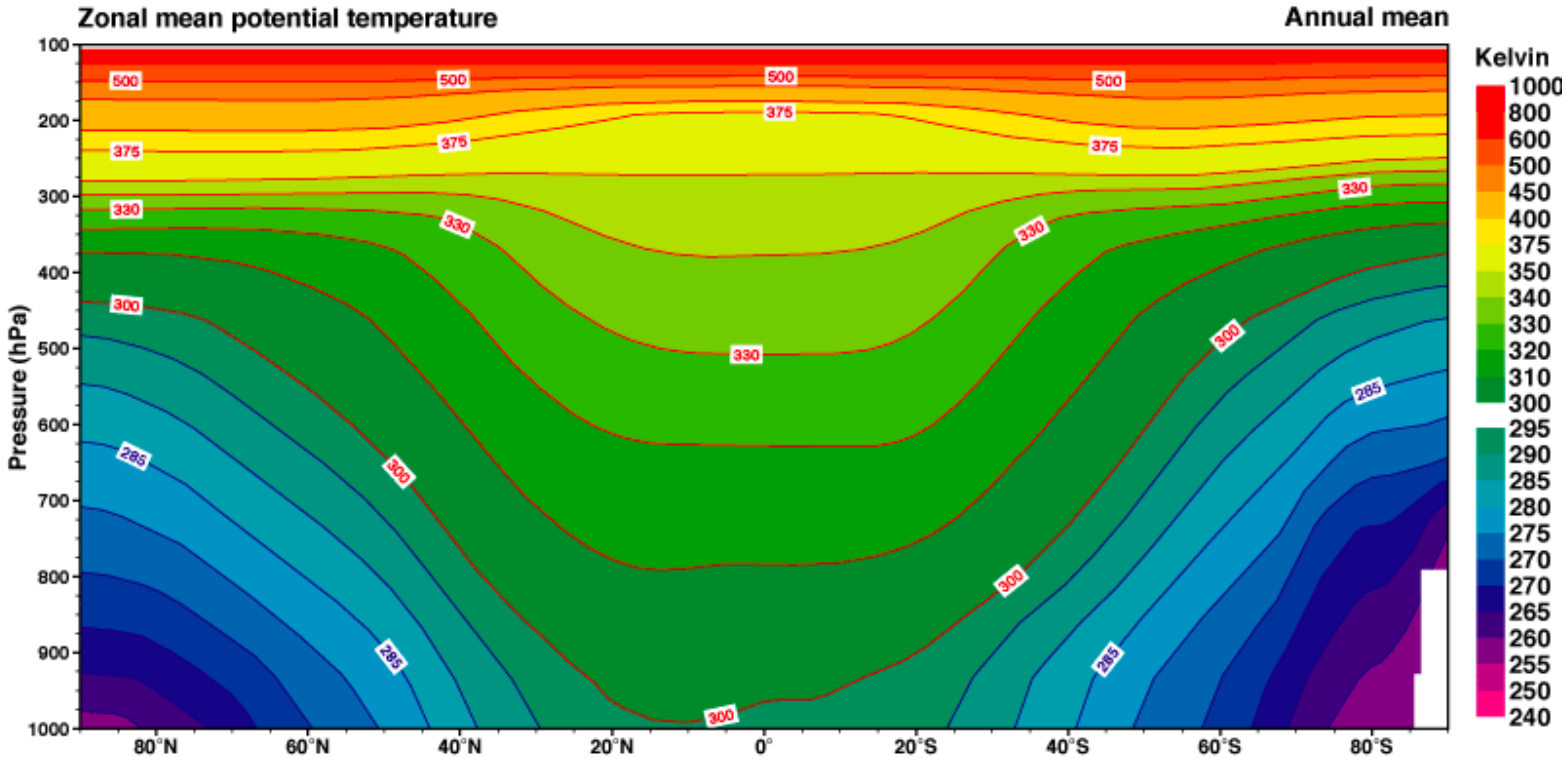
# Ertel Potential Vorticity

The deviation from WTG balance can then be defined using potential vorticity

$$\mathrm{PV} \simeq g\zeta S_p$$

However, WTG balance implies that the evolution of PV is determined by the evolution of vorticity.

$$\frac{\partial \mathrm{PV}}{\partial t} \simeq g S_p \frac{\partial \zeta}{\partial t}$$



# WTG deviation

Why are deviations from strict WTG balance non-divergent?

$$\mathbf{v}' \simeq \mathbf{k} \times \nabla_h \psi'$$

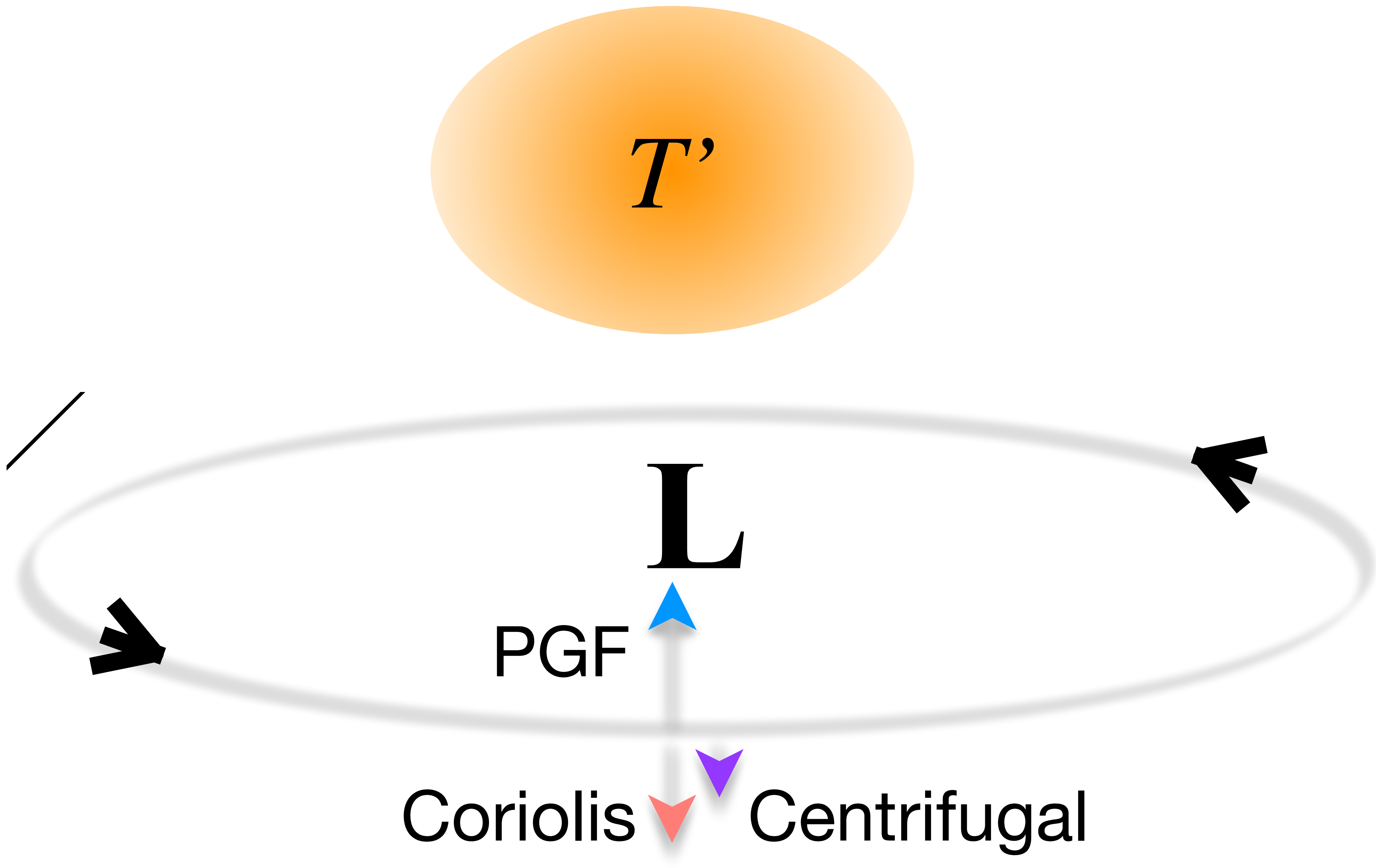
Balanced non-divergent circulations require pressure gradients to exist.

If the circulation is associated with heating, then a temperature anomaly must also exist: a deviation from strict WTG balance.

**Tropics**

*Nonlinear balance*

**Gradient wind,**
**Cyclostrophic**
**Semi-geostrophic**



# WTG PV equation

The WTG vorticity equation is:

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left( \mathbf{v}\zeta_a - \omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p} \right)$$

Horizontal absolute vorticity flux

"Tilting-like effect"

Note that the term on the rhs can be interpreted as a horizontal flux

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \mathbf{Z}_h$$

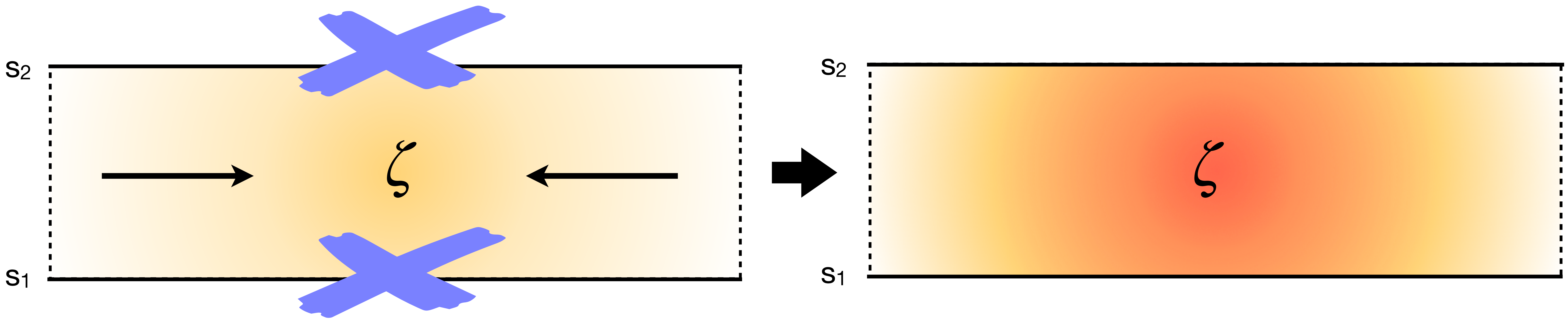
Note that the term on the rhs can be interpreted as a horizontal flux

# WTG PV equation

**Consequence:**

WTG balance does **NOT** allow $\zeta$ to be fluxed vertically.

$$\frac{\partial \zeta}{\partial t} = -\nabla_h \cdot \left( \mathbf{v}\zeta_a - \omega_w \mathbf{k} \times \frac{\partial \mathbf{v}}{\partial p} \right)$$




**You can only accrete vorticity within single pressure level**

# Convection under WTG balance

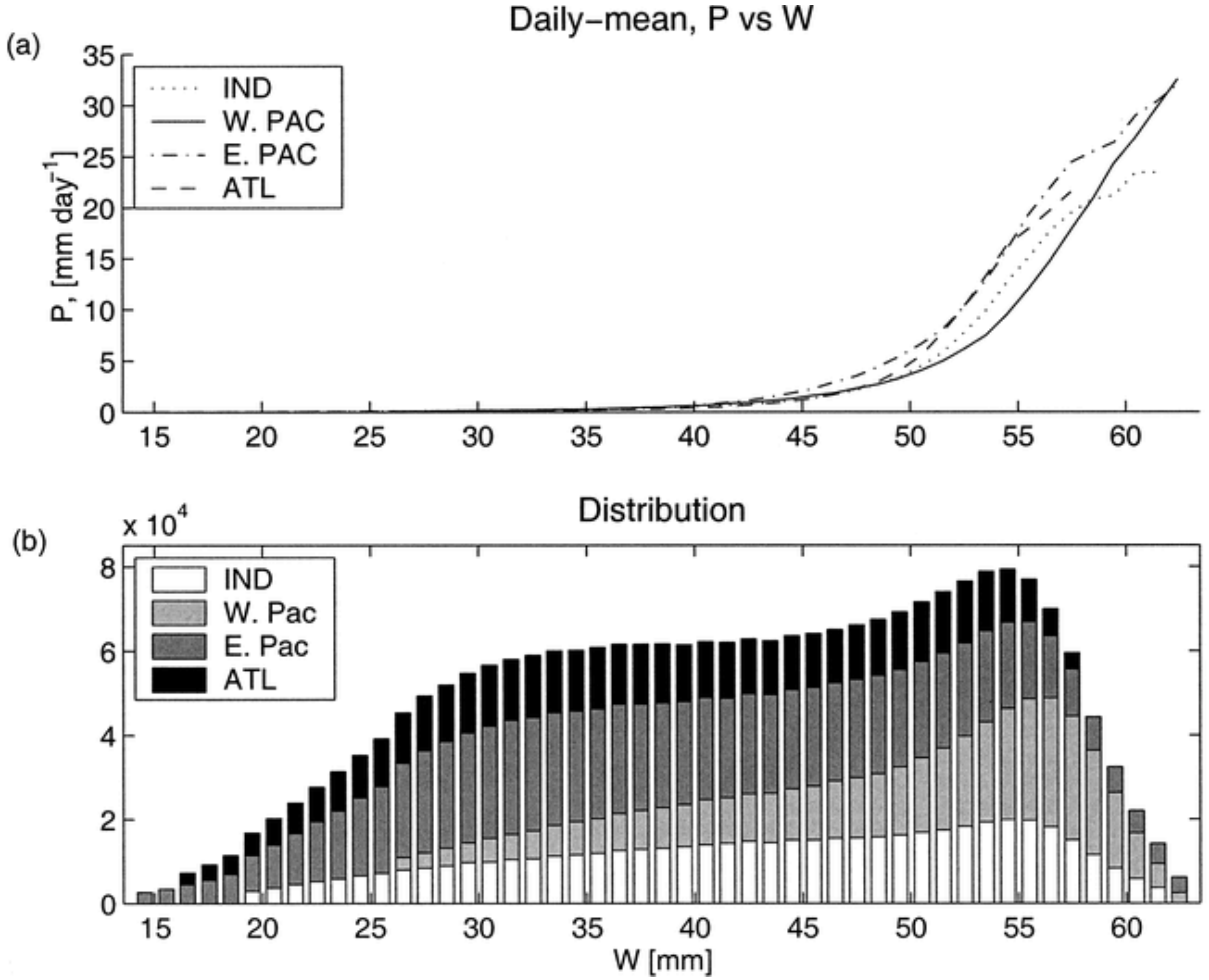
In L10 we found that:

$$L_v P = \frac{S_p L_v}{\mu_c^* m^4 \Delta p} \ln\left(\frac{T_{fl}}{T_{lnb}}\right) \nabla_h^2 \langle q \rangle - \langle Q_r \rangle$$

$$-\mathcal{O}\,\nabla_h^2 \langle q \rangle = L_v P + \langle Q_r \rangle$$

Under WTG we chan show that moisture acts as a velocity potential, we have

$$\mathcal{O}\,\nabla_h^2 \langle q \rangle = \nabla_h^2 \langle X_w \rangle S_p$$

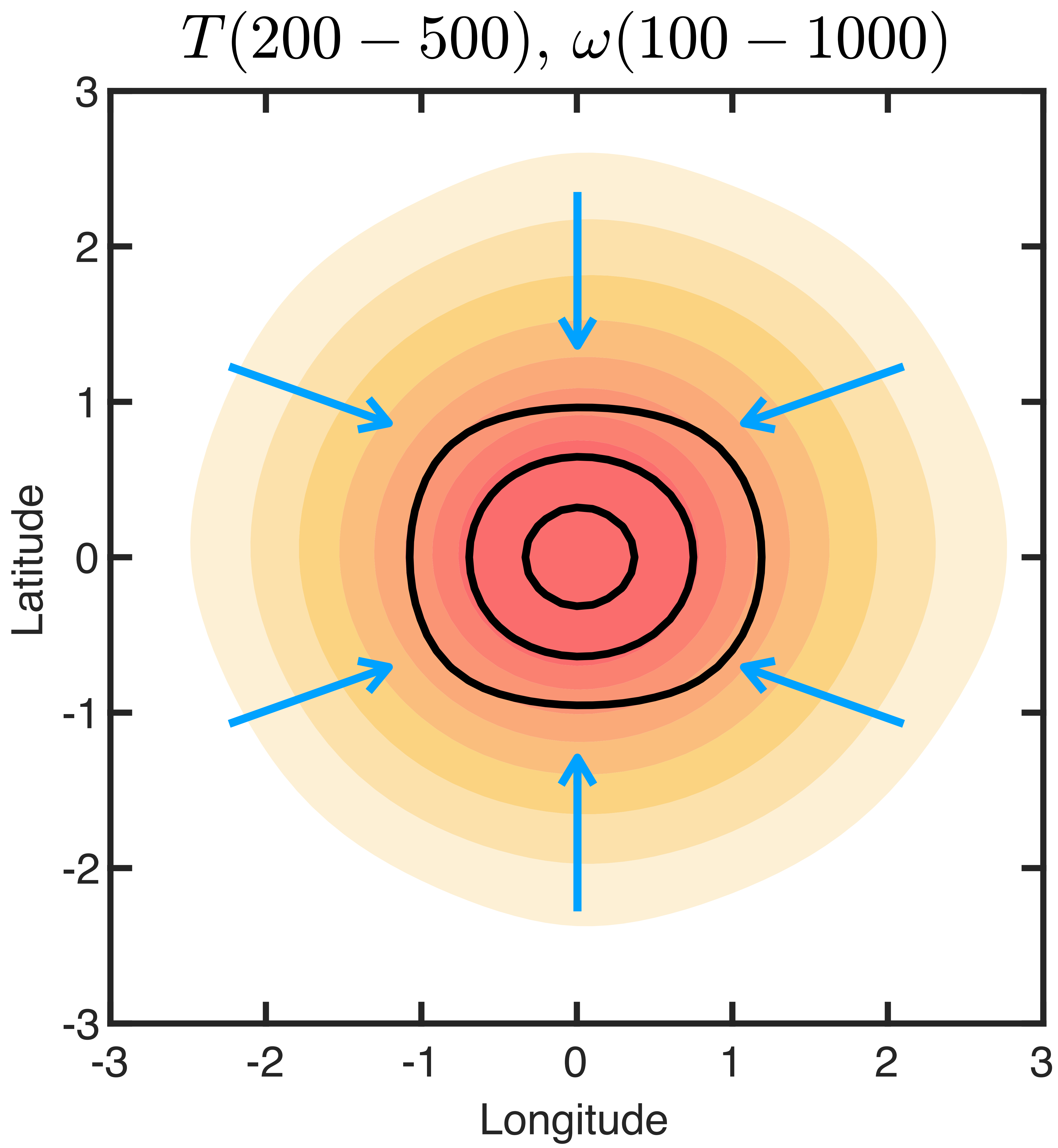



Bretherton, Peters, and Back (2004)

# Buoyancy/Moisture as a velocity potential

In mesoscale regions iof precipitation, we can interpret buoyancy/moisture as a velocity potential that satisfies

$$\langle X_w \rangle = \frac{\mathcal{O}}{S_p} \langle q \rangle$$



So what happens when moisture becomes important …