WTG adjustment and WTG applicability

$N_w = \max\left(\frac{c_p^2}{c^2}, \frac{L_y^2}{L_d^2}\right) \ll 1$ for WTG to apply

Note that $c = \frac{L_y}{T_g}$ $\qquad L_d = \frac{c}{f} = \frac{L_y}{T_g f}$

going to N_w definition and plugging yields:

$$N_w = \max\left(\frac{L_x^2}{\tau^2}\frac{T_g^2}{L_y^2}, \frac{\cancel{L_y^2}\, T_g^2 f^2}{\cancel{L_y^2}}\right)$$

If we assume that $L_x \sim L_y$ then

$$N_w = \max\left(\frac{T_g^2}{\tau^2}, T_g^2 f^2\right)$$

$$= T_g^2 \max\left(\tau^{-2}, f^2\right)$$

In Adames (2022) we see that in most tropical motions

$$Ro_\tau = \frac{1}{\tau f} \sim 1$$

$\therefore$ $\tau^{-2} \sim f^2$ so that $N_w \simeq \frac{c_p^2}{c^2} \simeq \frac{T_g^2}{\tau^2}$

Let's say that $N_w \sim 0.1$ for WTG balance to exist

$$\frac{T_g^2}{\tau^2} \sim 0.1 \rightarrow T_g^2 \sim \frac{\tau^2}{10}$$

You'll get that: $T_g \sim \frac{\tau}{3}$

If you use c_p then $c_p \sim \frac{c}{3} \sim \frac{50}{3} \simeq 17\ \mathrm{ms}^{-1}$

Tropical dynamics under WTG balance

When $N_w \ll 1$ we can decompose the wind as:

$$\vec{u} = \vec{u_w} + \vec{u'}$$

strict WTG deviation from WTG

The WTG wind $\quad \omega_w \frac{\partial DSE}{\partial p} = Q_1$ when $N_w \ll 1$

$$-\omega_w S_p = Q_1 \qquad S_p = -\frac{\partial DSE}{\partial p}$$

positive no.

$\vec{v_w} = \nabla_h \chi_w$ WTG velocity potential

Mass cont. $\quad \nabla_h \cdot \vec{v_w} = -\frac{\partial \omega_w}{\partial p}$ plug in χ_w

$$\nabla_h^2 \chi_w = -\frac{\partial \omega_w}{\partial p}$$

Define $X_w \rightarrow$ integrated vel. potential

$$\omega_w = \nabla_h^2 X_w$$

$$\chi_w = -\frac{\partial X_w}{\partial p}$$

The WTG circulation can be completely described by X_w which follows $\quad -S_p \nabla_h^2 X_w = Q_1$

Helmholtz theorem

$$\vec{v} = \nabla_h \chi + \hat{k} \times \nabla_h \psi$$

Under WTG balance $\chi \simeq \chi_w$

It follows that $\psi \simeq \psi'$

$$\vec{v} \simeq \underbrace{\nabla_h \chi_w}_{\text{strict WTG wind}} + \underbrace{\hat{k} \times \nabla_h \psi'}_{\text{deviation from strict WTG}}$$

which means that deviations from WTG can be explained using the PV equation.

The WTG component is described by S_w or ω_w