AOS 801: Advanced Tropical Meteorology

Lecture 13   Spring 2023

Tropical Dynamics Under WTG Balance

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When is WTG balance valid?

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$

Given that there's an adjustment process associated with gravity waves, we must assume systems that are in WTG balance must evolve slowly.

But **how slowly**? We must invoke scale analysis to find out!

WTG adjustment timescale

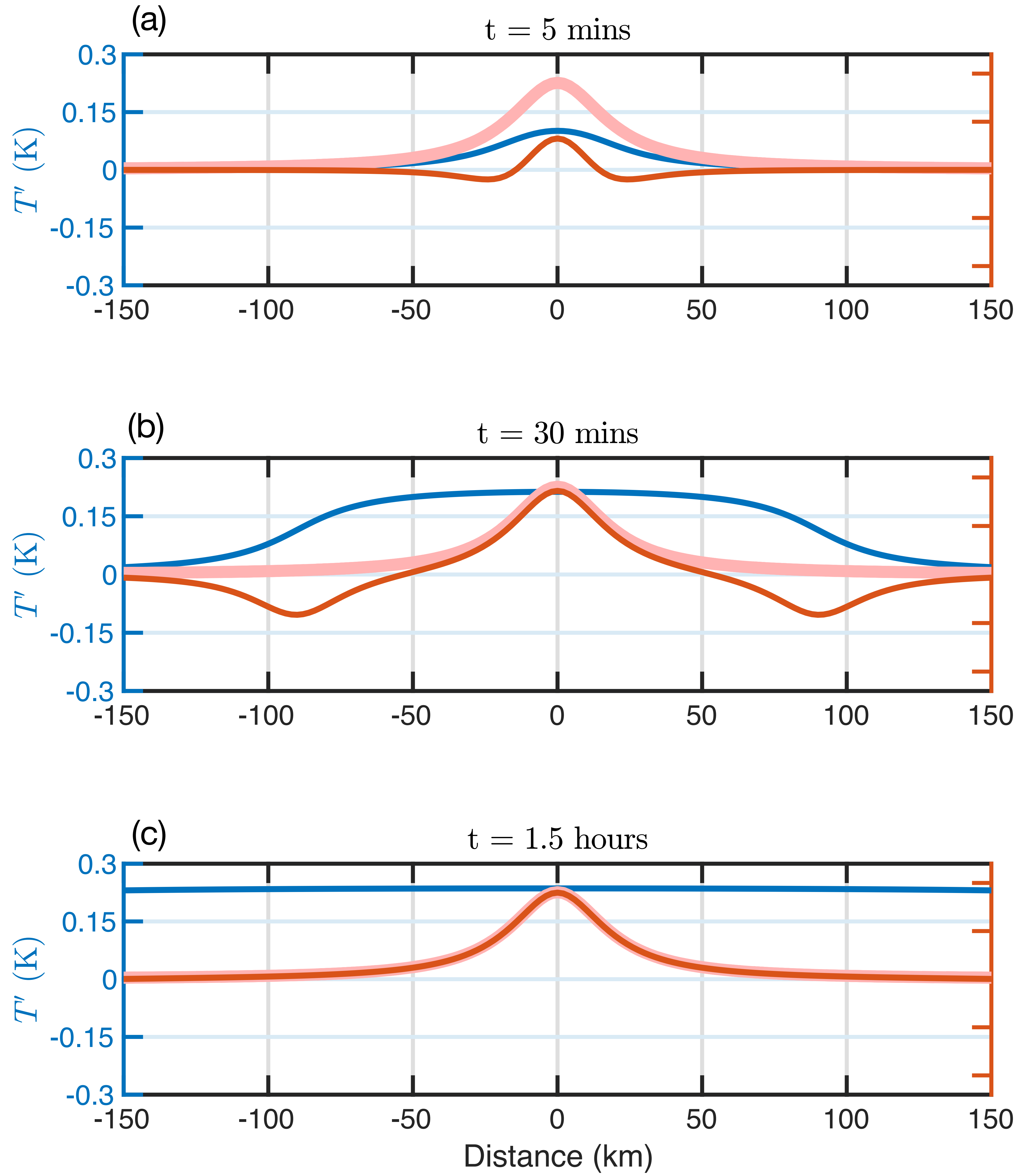
In the case of fixed heating, WTG is attained when the gravity waves exit the region of interest.

This is known as the **gravity wave adjustment timescale**

$$\tau_g = \frac{L}{c}$$

L = domain size

c = gravity wave phase speed



Adames and Maloney (2021)

Basic equations

In order to understand when and why WTG is valid, we must scale the entire set of basic equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla_h)\mathbf{v} + \omega \frac{\partial \mathbf{v}}{\partial p} = -f\mathbf{k} \times \mathbf{v} - \nabla_h \Phi$$

Horizontal Momentum

$$\frac{\partial \Phi}{\partial p} = -\frac{R_d T}{p}$$

Hydrostatic

$$\frac{\partial \omega}{\partial p} = -\nabla_h \cdot \mathbf{v}$$

Mass Continuity

$$\frac{\partial C_p T}{\partial t} + \mathbf{v} \cdot \nabla_h C_p T + \omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$

Thermodynamic

$$\frac{\partial L_v q}{\partial t} + \mathbf{v} \cdot \nabla_h L_v q + \omega \frac{\partial L_v q}{\partial p} = -Q_2$$

Moisture

Scale analysis

You will read Adames (2022) (uploaded to canvas) to understand the scaling. It will also be your HW3.

The non-dimensional thermodynamic (DSE) budget is written as:

$$\mathrm{N}_w\left(\frac{\partial \hat{\mathrm{DSE}}}{\partial \hat{t}} + \frac{\mathrm{U}}{\mathrm{c}_p}\hat{\mathbf{v}}\cdot\hat{\nabla}\hat{\mathrm{DSE}}\right) - \hat{\omega}\hat{S}_p = \hat{\alpha}\hat{Q}_c + (1-\hat{\alpha})\hat{Q}_r \qquad S_p = -\frac{\partial \mathrm{DSE}}{\partial p}$$

Where:

$\mathrm{N}_w = \max\left(\mathrm{Fr}_\tau^2, \mathrm{Bu}^{-1}\right)$ **WTG number**

$\mathrm{Fr}_\tau \equiv \frac{\mathrm{c}_p}{\mathrm{c}}$ **Froude Number** c_p is the propagation speed of the system

$\mathrm{Bu} \equiv \left(\frac{\mathrm{L}_d}{\mathrm{L}_y}\right)^2$ **Burger Number**

$\mathrm{L}_d = \frac{\mathrm{c}}{f}$ **Rossby radius of deformation**

Conditions for WTG balance

From examination of the non-dimensional thermodynamic equation we see that $\mathrm{N}_w \ll 1$ for WTG balance to be valid:

$$\mathrm{N}_w = \max\left(\frac{c_p^2}{c^2}, \frac{L_y^2}{L_d^2}\right) \ll 1$$

Condition 1: The system must evolve much more slowly than free gravity waves.

Condition 2: The meridional scale of the system must be smaller than the Rossby radius of deformation.

What do these mean physically?

Exercise:

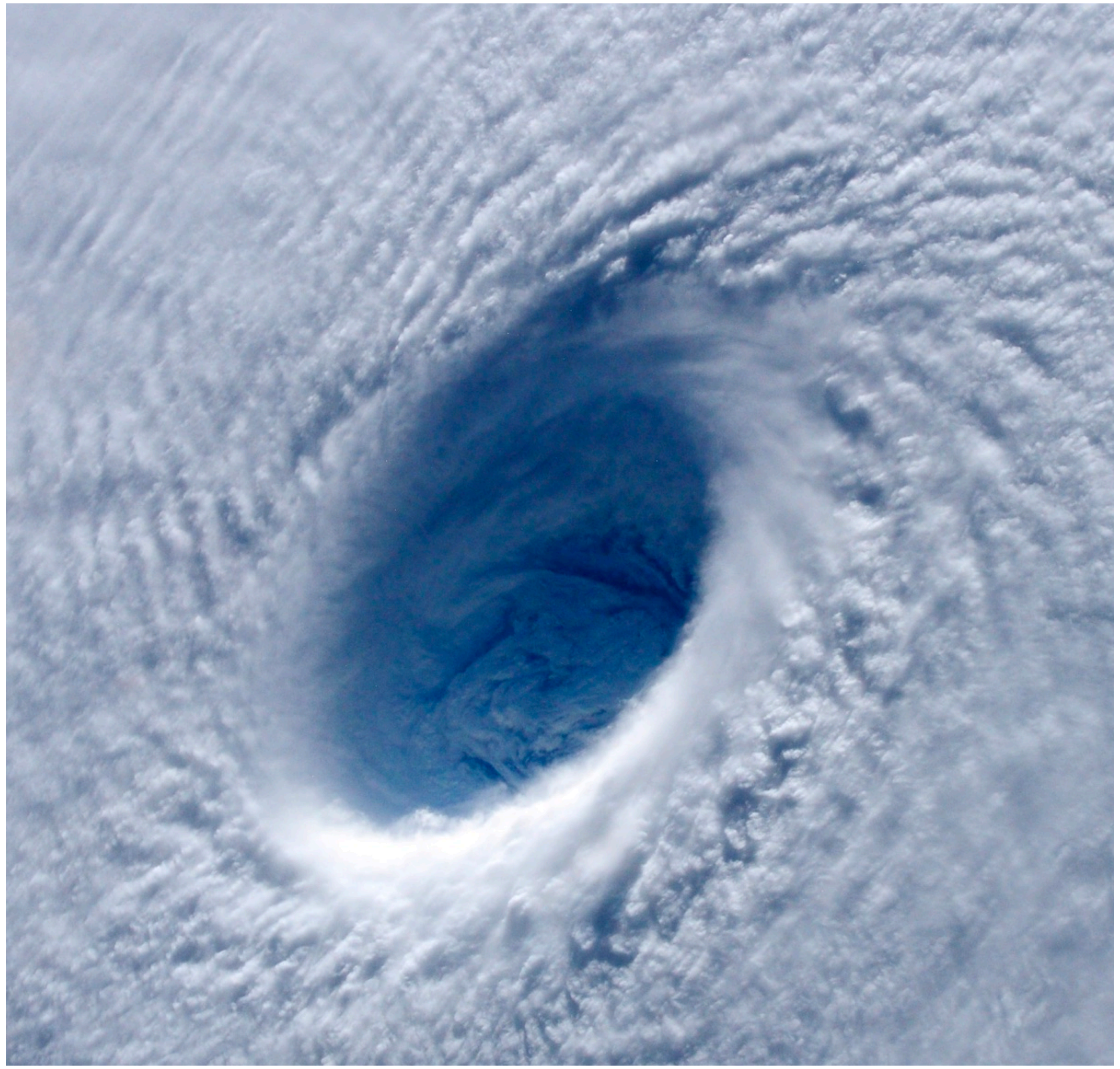
Is a mature tropical cyclone at 15°N in WTG balance?

$$\mathrm{N}_w = \max\left(\frac{c_p^2}{c^2}, \frac{L_y^2}{L_d^2}\right) \ll 1?$$

Assume the TC moves at 5 m/s, has a first baroclinic mode in vertical motion, a meridional scale of 300 km and the TC deformation radius is given by:

$L_d = \dfrac{c}{\zeta_a}$ where $\zeta_a = \zeta + f$ is the absolute vorticity

Assume that $\zeta = U/L_y$ and $U \sim 30$ m/s. Give a physical explanation to your answer.



What about an easterly wave?

Is an easterly wave at 10°N in WTG balance?

$$\mathrm{N}_w = \max\left(\frac{c_p^2}{c^2}, \frac{L_y^2}{L_d^2}\right) \ll 1?$$

Assume the wave moves at 10 m/s, has a first baroclinic mode in vertical motion, a meridional scale of 300 km (wavelength = $\lambda = 2\pi L_y$)

$L_d = \dfrac{c}{f}$. Assume t d $U \sim 1$ m/s. Give a physical explanation to your answer.