Conditions where WTG balance is met.

Nondimensional thermodynamic budget

$$N_W \left(\frac{\partial \widehat{DSE}}{\partial \hat{t}} + \frac{U}{C_p} \hat{v} \cdot \hat{\nabla}_h \widehat{DSE} \right) - \hat{\omega} \hat{S}_p = \hat{\alpha} \hat{Q}_c - (1-\hat{\alpha}) \hat{Q}_r$$

WTG scaling number

deviation from WTG balance

WTG-balanced component of Thermo budget

$U \sim$ zonal wind scale

$C_p \sim$ phase speed of disturbance

$C \sim$ gravity wave phase speed

$\hat{\alpha} \sim -\frac{(\partial L\overline{q} / \partial p)}{(\partial \overline{DSE} / \partial p)}$ Chikira parameter

$\alpha \sim RH$ — average of the column

$\alpha \sim 1$ saturated

$\alpha \sim 0$ dry

$$N_W = \max \left(\frac{C_p^2}{C^2}, \frac{L_y^2}{L_d^2} \right)$$

Froude Number $Fr^2 = \frac{C_p^2}{C^2}$

Inverse of the Burger number. $Bu := \frac{L_d^2}{L_y^2}$

$L_y \sim$ meridional scale of system

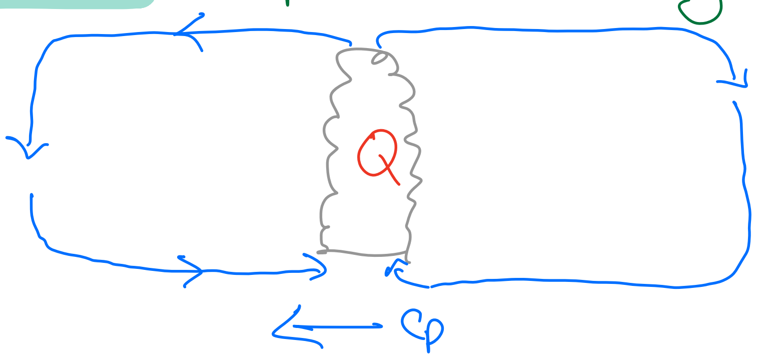
$L_y \sim \frac{\lambda_y}{2\pi}$

$L_d \sim$ Rossby deformation rad $= \frac{C}{f}$ ← planetary vort.

For WTG balance to be satisfied $N_W \ll 1$

Two conditions must be satisfied for $N_w << 1$

Condition 1 $\quad c_p^2 << c^2 \quad$ Why? $\quad \tau_g = \frac{L}{c}$



If you move too fast, you can't adjust to WTG

$$Fr^2 = \frac{c_p^2}{c^2} = \frac{\cancel{L_x^2}\,\tau_g^2}{\tau^2 \cancel{L_x^2}} = \frac{\tau_g^2}{\tau^2} << 1$$

$\boxed{\tau^2 >> \tau_g^2}$ System must evolve **more slowly** than the time it takes **gravity waves** to adjust domain towards **WTG balance**.

$\tau^2 \sim 10\tau_g^2$

$\tau \sim \sqrt{10}\,\tau_g$

$\boxed{\tau \sim 3\tau_g}$

Timescale of the system must be 3 times longer than gravity wave adj. timescale.

Example: $\tau_g = \frac{L_x}{c} = \frac{10^6\,m}{50\,ms^{-1}}$

$$\tau_g^2 = \frac{10^{12}\,m^2}{2.5\times10^3\,m^2s^{-2}} = 4\times10^8$$

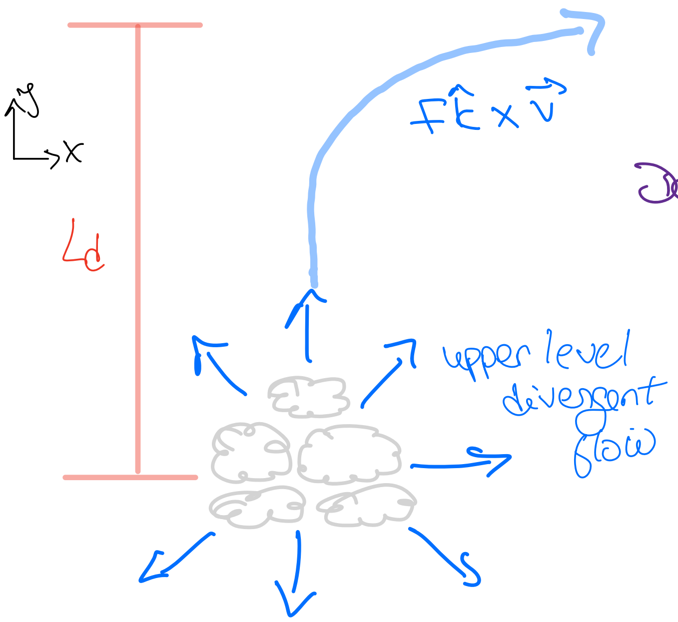
$\tau_g \simeq 2\times10^4\,s$

~ 6 hours

$\tau = \frac{L_x}{c_p} = \frac{10^6\,m}{5\,ms^{-1}}$ $\qquad \tau = \frac{10^{12}}{2.5\times10^1} = 4\times10^{10}$

$\tau \sim 2 \times 10^5$ s
$\sim$ 2+ days

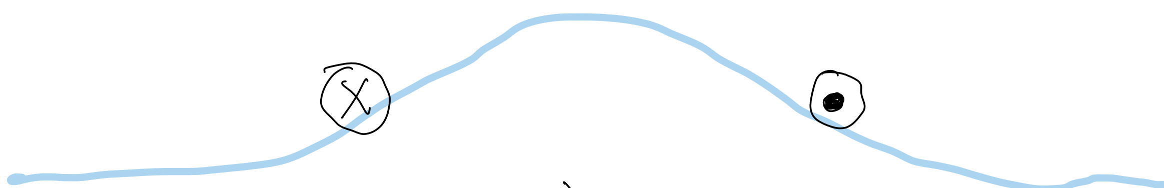
Condition 2 Meridional scale is smaller than deformation radius.



$f \sim f_0$
$f\hat{k} \times \vec{v}$

Deformation radius is a distance it takes for the Coriolis force to become dominant in the mom. eqn

If your system has a scale $L_d \sim L_y$ then



Coriolis force won't allow WTG balance

$$L_d \sim \frac{c}{f} = \frac{50\ ms^{-1}}{3 \times 10^{-5} s^{-1}} \approx \frac{5}{3}\ \frac{10}{10^{-5}} = \frac{5}{3} \times 10^6$$

$$L_y \sim \frac{\lambda}{2\pi} = \frac{10^6 m}{6} = \frac{5}{3} \times 10^5$$