Forced wave equation:

$$\frac{\partial \vec{v}'}{\partial t} = -\nabla_h \Phi' \quad (1) \qquad \frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial P}\right) = -w'\sigma - Q$$

Thickness ... heating

$$\nabla_h \cdot \vec{V}' = -\frac{\partial w'}{\partial P} \quad (2) \qquad \frac{\partial \Phi'}{\partial P} = -\frac{R_d T'}{P} \qquad \sigma = \frac{R_d S_p}{P C_p}$$

From here on, we follow the procedure from last class, and obtain a wave equation with Q:

$$\frac{\partial^4 \Phi'}{\partial P^2 \partial t^2} = -\sigma \nabla_h^2 \Phi' + \frac{\partial Q}{\partial t}$$

Forced wave equation

Soluable if we know what Q is

Let's assume that $\partial^2/\partial p^2 = -m^2$

$$\frac{\partial^2 \Phi}{\partial t^2} = \frac{\sigma}{m^2}\nabla_h^2 \Phi - \frac{1}{m^2}\frac{\partial Q}{\partial t} \qquad c^2 = \frac{\sigma}{m^2}$$

$$= c^2 \nabla_h^2 \Phi + \frac{\partial Q^*}{\partial t} \qquad Q^* = -\frac{Q}{m^2}$$

Assume 1-D and we get

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \frac{\partial^2 \Phi}{\partial x^2} = \frac{\partial Q^*}{\partial t}$$

Reduced forced wave eqn.

What if $Q^* = F(x) H(t)$ $\qquad H = 1 \quad t \geq 0$

$\qquad H = 0 \quad t < 0$

$$\frac{\partial H(t)}{\partial t} = \delta(t)$$

Dirac Delta Function

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \qquad \delta(t \neq 0) = 0$$

$$\delta(t = 0) = \infty$$

The forced wave eqn. becomes

$$\frac{\partial^2 \Phi}{\partial t^2} - c^2 \frac{\partial^2 \Phi}{\partial x^2} = \delta(t) F(x) \qquad (1)$$

Eq. (1) has a solution in the form of a Green's function.

Known solution to the forced wave eqn. (Eq. 1)