AOS 801: Advanced Tropical Meteorology

Lecture 11   Spring 2023

Adjustment towards Weak Temperature Gradient Balance

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Announcements

Let's finish the paper discussion.

HW2 is due March 6.

Wolding et al. (2022)

Draw a schematic of a convective life cycle and point out the main findings of Wolding et al. (2022)

How do the results compare to what you have learned in class?

Recap

Up to this point we have discussed the thermodynamics of the tropics and tropical convection.

While individual clouds are on the order of 1-100 km across, they have a massive impact on the large-scale.

Let us know think about what comes next

Division of course topics

✓ **Introduction:** Quick review of the equations of motion, scale analysis, a gentle introduction to the WTG approximation.

✓ **Tropical deep convection** Review of moist thermodynamics, quasi-equilibrium principles, convection organization, instabilities.

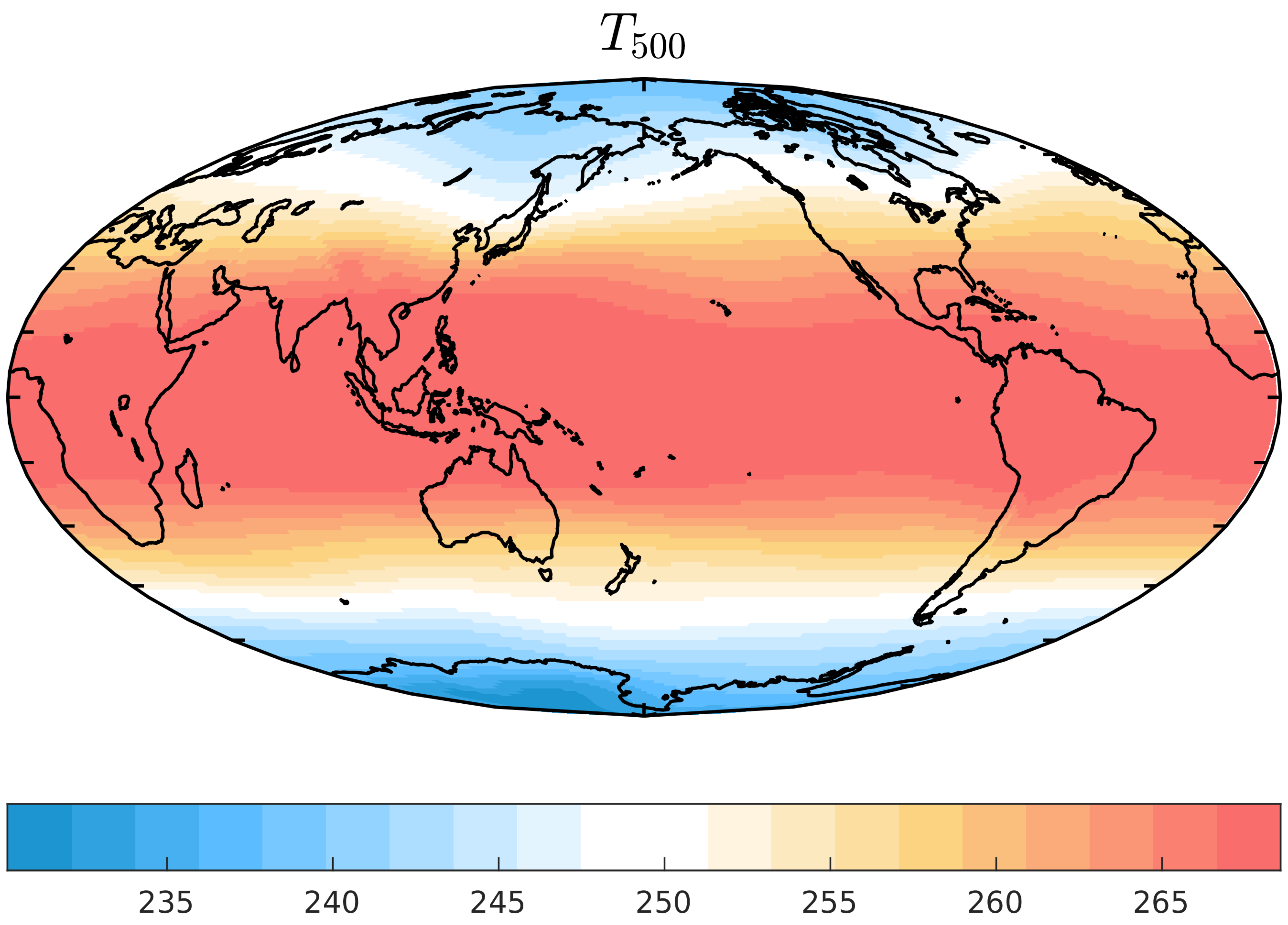
Large-scale tropical circulations: WTG, large-scale tropical waves, the MJO.

Tropical Cyclones: formation, steady state, intensification, movement, role in tropical circulation.

The weak temperature gradient (WTG) approximation

The mean temperature distribution in the equatorial belt 20°N/S is smooth.

But how do we get to this stage in the first place.

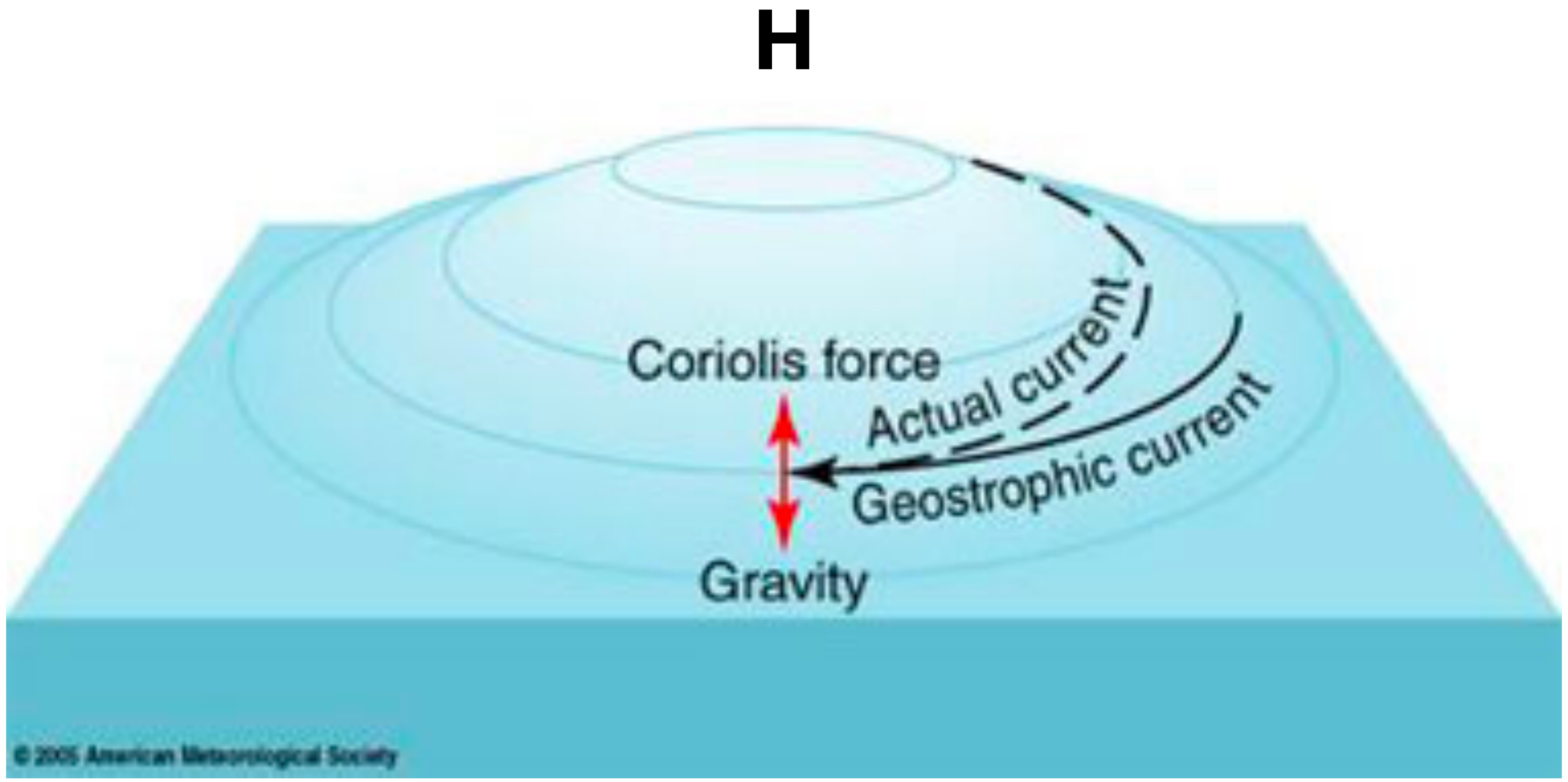


The midlatitudes

In the midlatitudes, flow coming out of a high gets deflected by the Coriolis force, eventually reaching geostrophic balance.

$$-fv_g = -\frac{\partial \Phi}{\partial x}$$ u-momentum

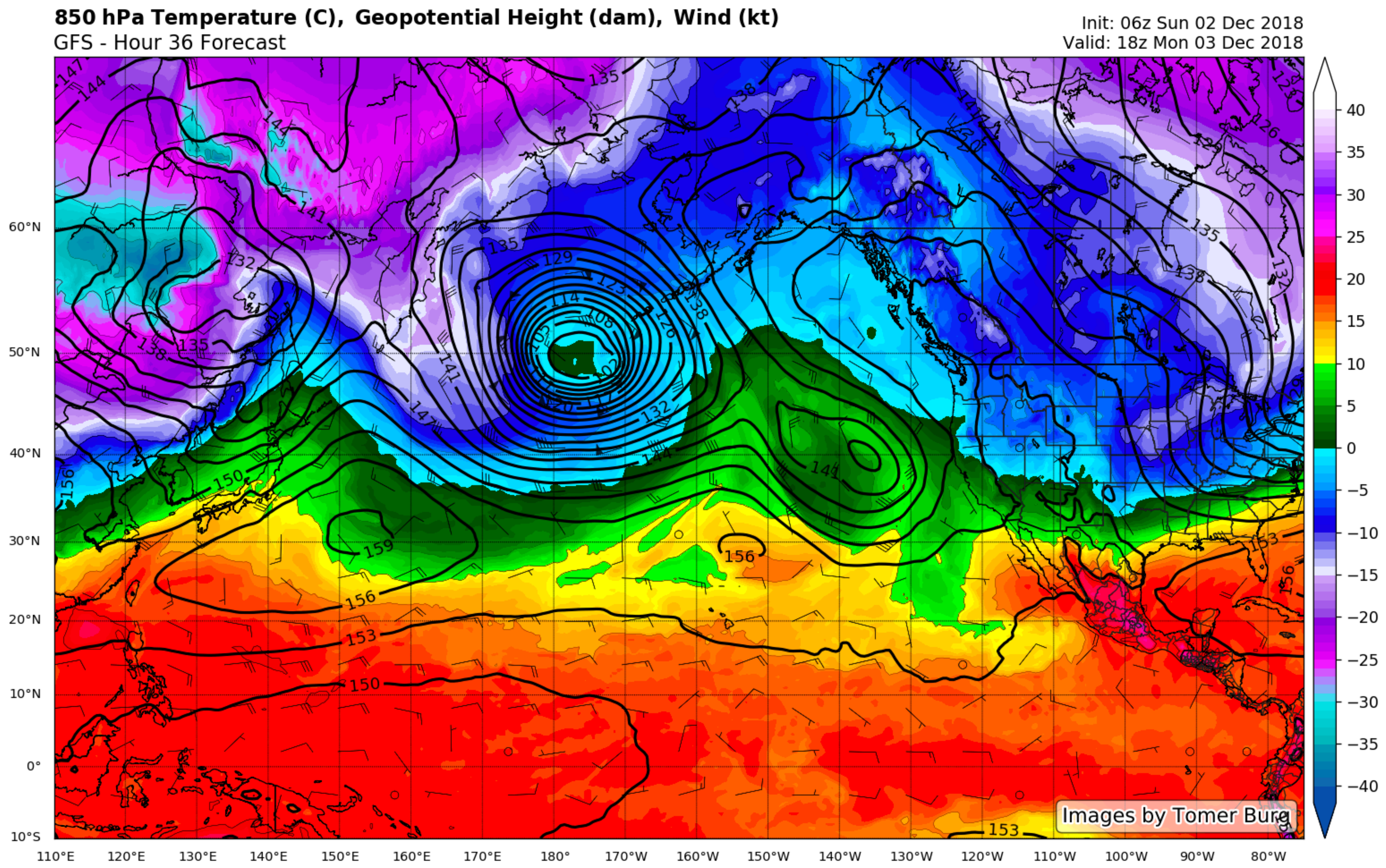
$$fu_g = -\frac{\partial \Phi}{\partial y}$$ v-momentum



The tropics

Pressure gradients in the tropics are much smaller than in the midlatitudes.

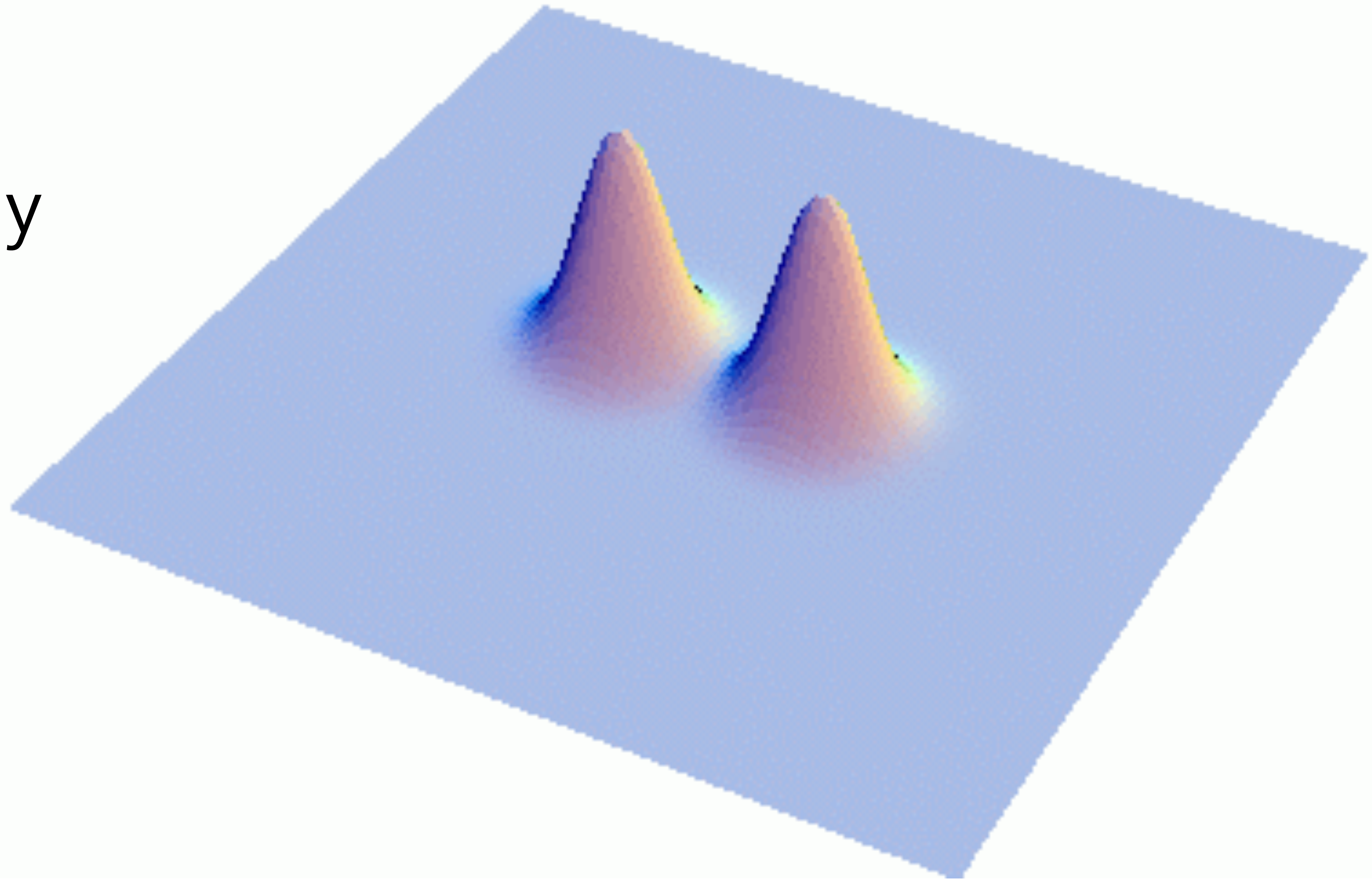
Exception: TCs



Gravity waves in the tropics

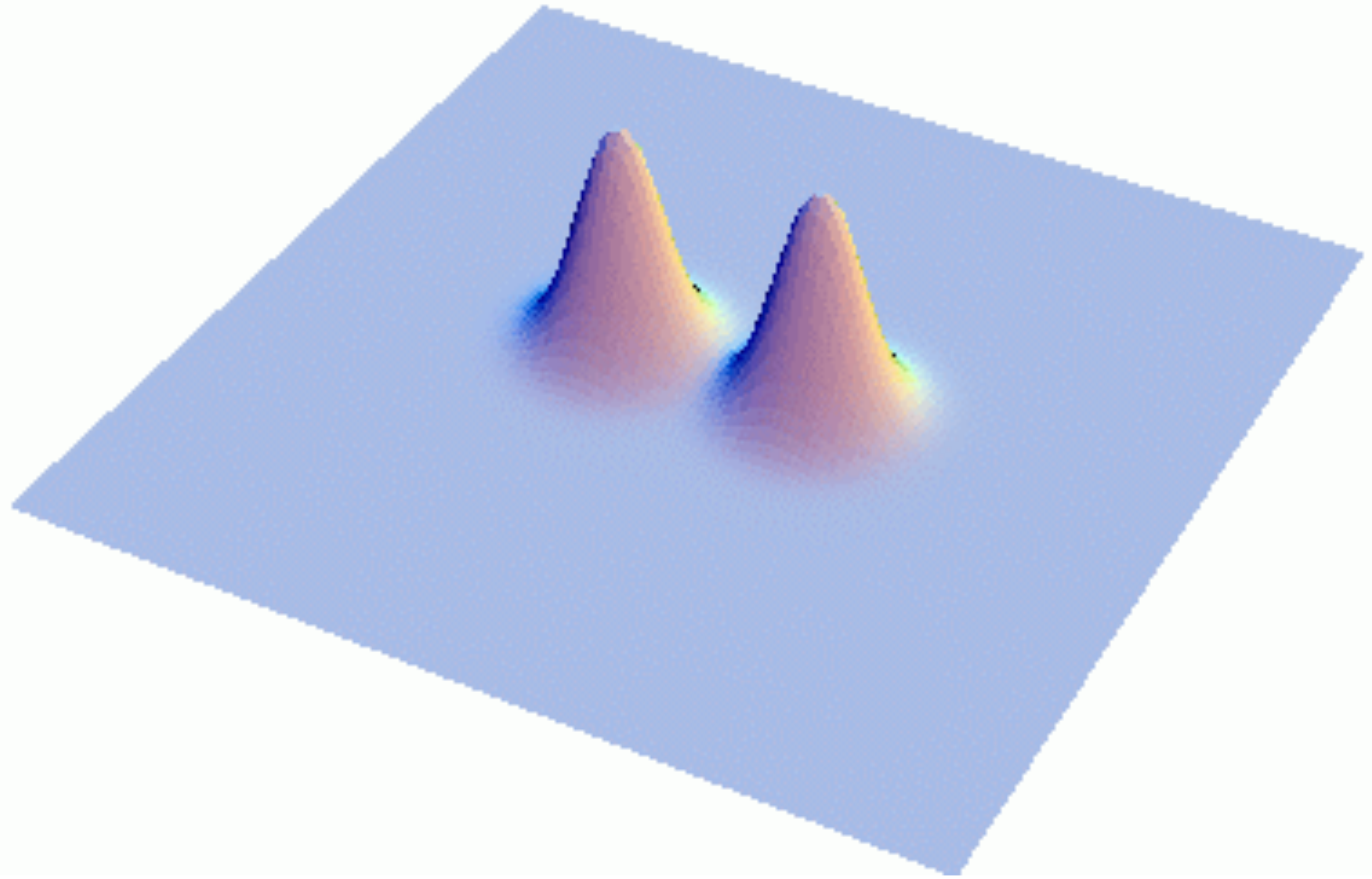
As planetary vorticity goes down to zero, geostrophic balance ceases to occur.

Instead, the flow spreads away horizontally in the form of gravity waves.



Gravity waves

Let's assume that you have zero Coriolis force , no advection and no heating. The equations are written as:



$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla_h \Phi'$$ **Momentum**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$ **Mass**

$$\frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial p}\right) = -\omega\sigma$$ **Thermo + Hydrostatic**

$$\sigma = \frac{R_d S_p}{p C_p}$$ **Static stability parameter**

The gravity wave equation in isobaric coordinates

Combining the three equations yields the gravity wave equation

$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial p}\frac{1}{\sigma}\frac{\partial^2}{\partial t \partial p}\right)\Phi' = -\nabla_h^2 \Phi'.$$

Assuming a wave solution of the form:

$$\Phi' = \hat{\Phi}\exp\left(ikx + ily + imp - i\varpi t\right)$$

Yields the following phase speed:

$$c = \frac{\varpi}{K} = \pm\frac{\sqrt{\sigma}}{m}, \qquad \text{where } K^2 = k^2 + l^2$$

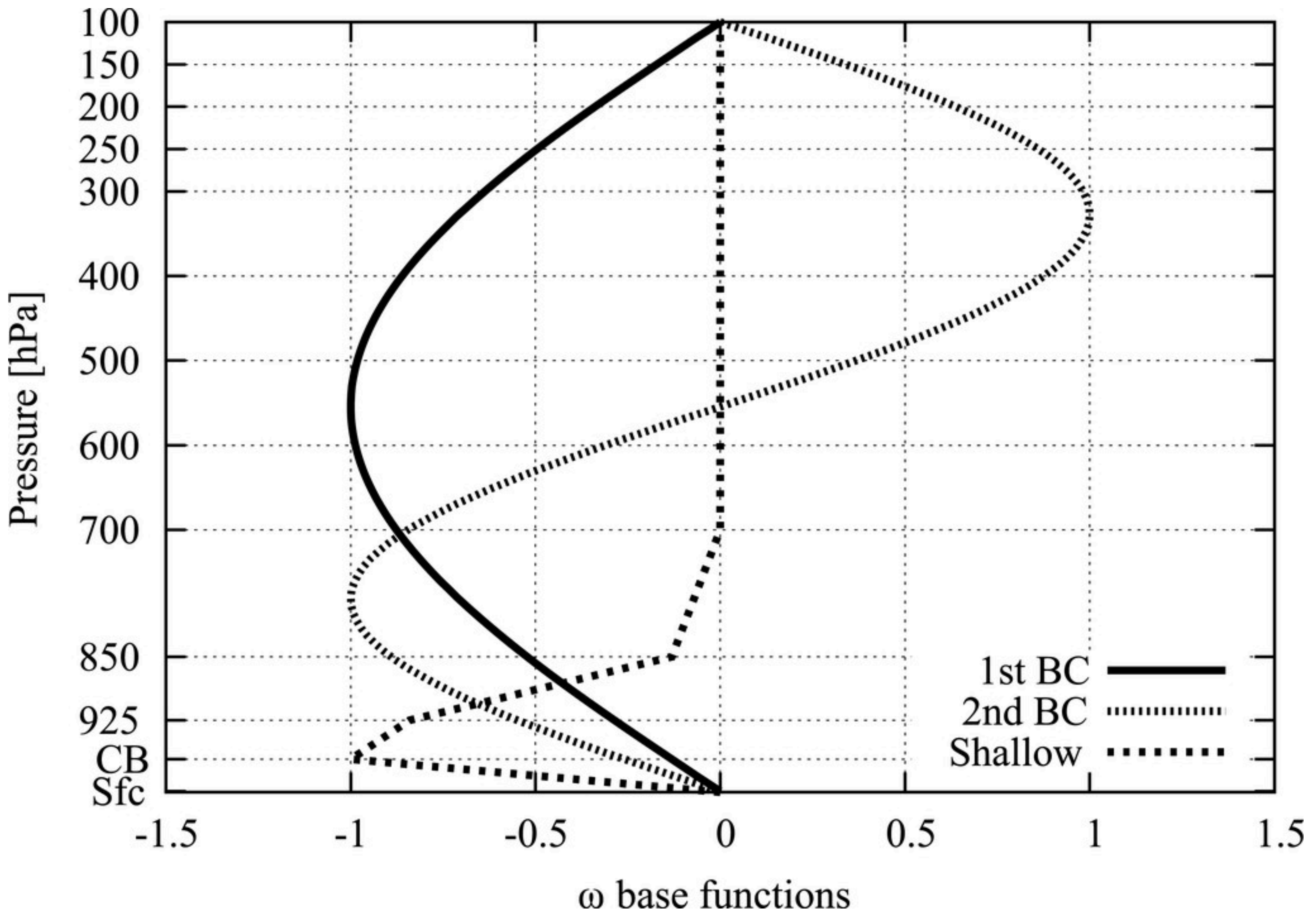
What determines the gravity wave speed?

The phase speed is :

$$c = \pm \frac{\sqrt{\sigma}}{m}$$

Plugging realistic numbers onto c yields a value of 50 m/s for the first baroclinic mode.

50 m/s = 112 mph, you can't outdrive this wave.

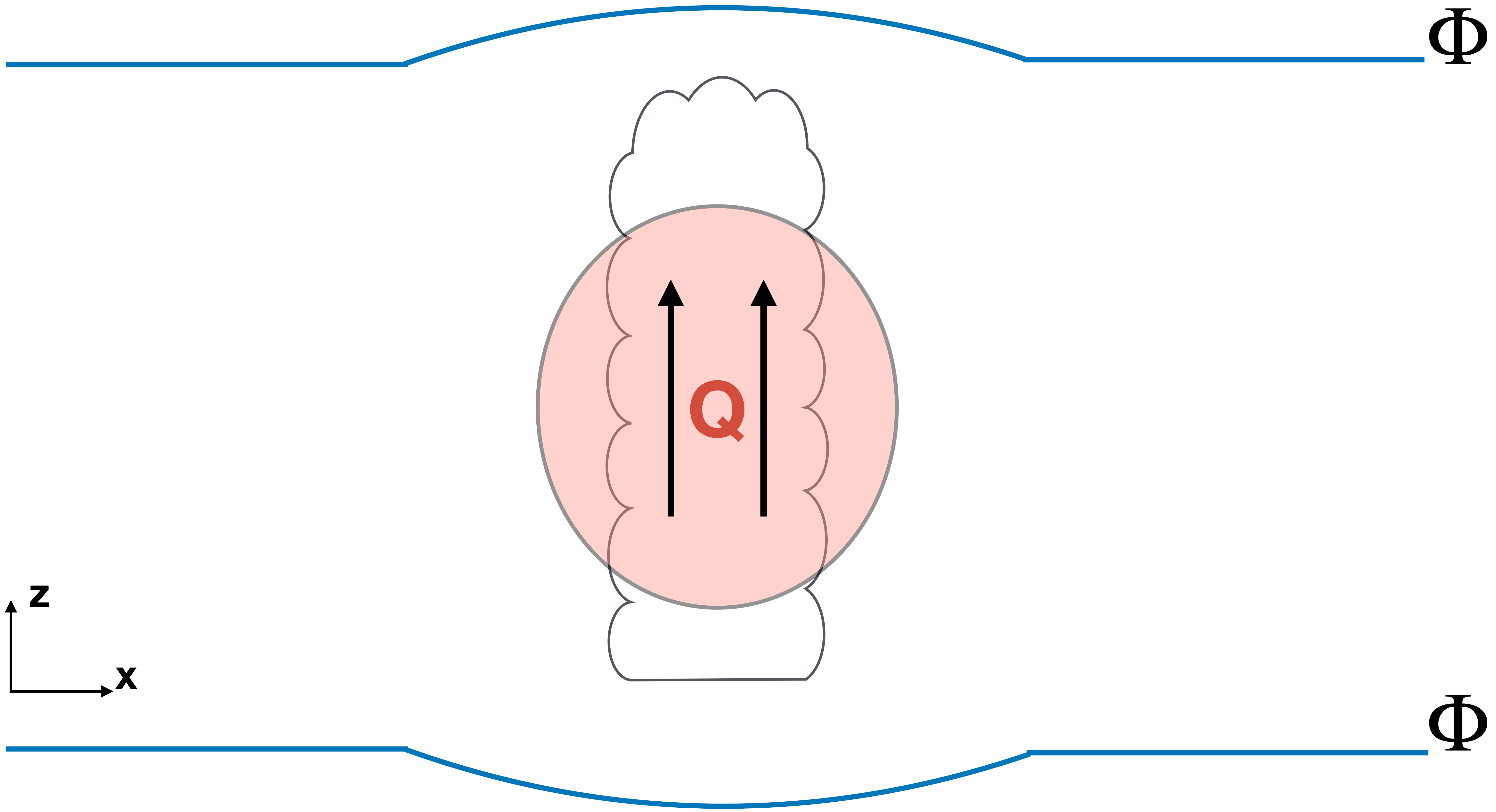


Masunaga and L'Ecuyer (2014)

Weak temperature gradient balance

A region of strong convection has a lot of latent heat release from condensation, warming up the cloud. Geopotential increases in the upper troposphere and decreases in the lower troposphere to adjust to hydrostatic balance.

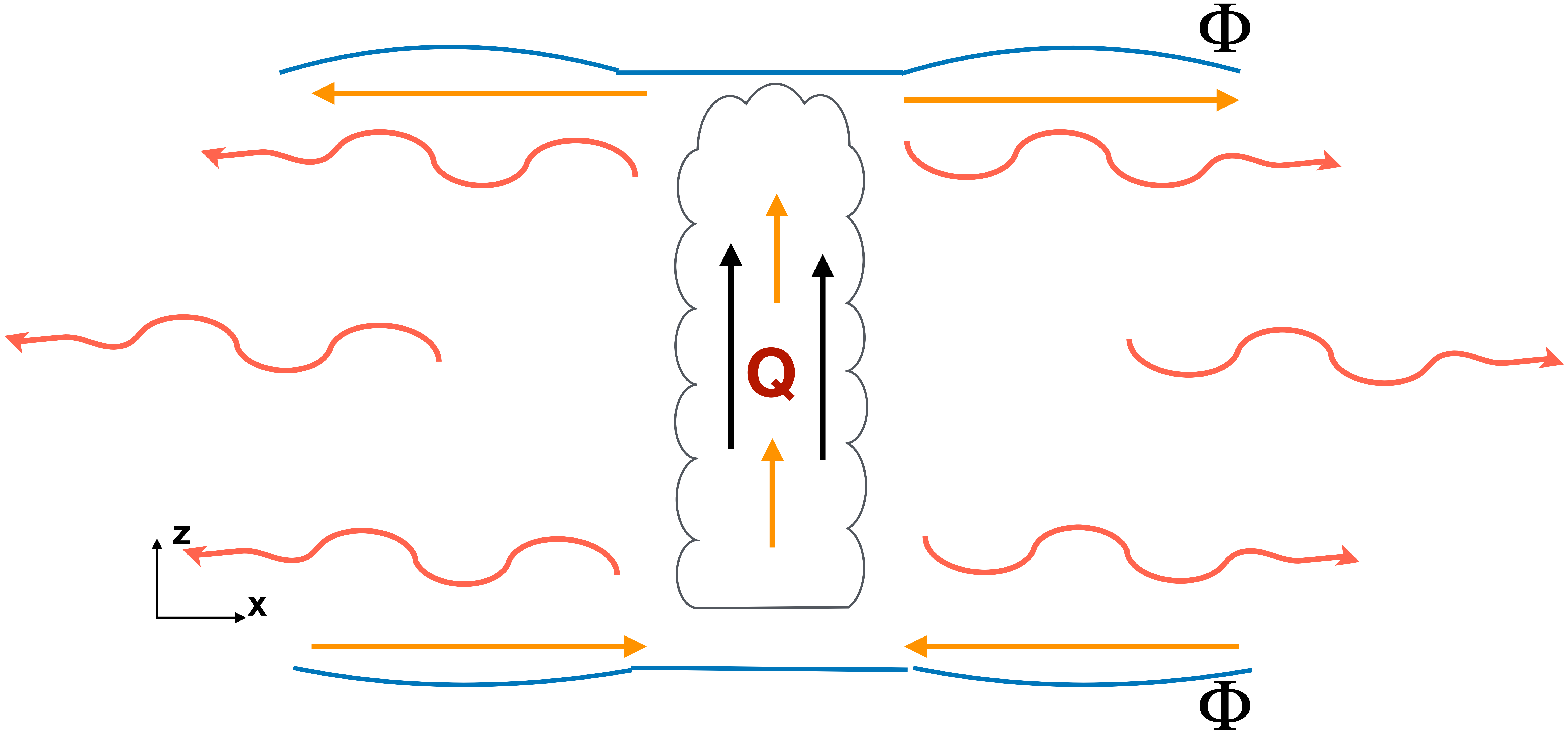
$$\omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$



Weak temperature gradient balance

Gravity waves develop from the convection and "smooth out" the geopotential/ temperature anomalies. A *secondary* circulation develops from the gravity waves, which adds further upward motion to the convection, cooling the cloud.

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$



Weak temperature gradient balance

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} = Q_1$$

$$\mathrm{DSE} = C_p T + \Phi$$

This process redistributes entropy (warm air has higher entropy)

Weak temperature gradient balance

Let's return to the equations that gave us gravity waves and add a heat source $\mathcal{Q}$ (i.e. a mass source). For simplicity, let's consider one dimension:

$$\frac{\partial u'}{\partial t} = -\frac{\partial \Phi'}{\partial x}$$

$$\frac{\partial \Phi'}{\partial t} + c^2 \frac{\partial u'}{\partial x} = \mathcal{Q}$$

Where we have now defined the gravity wave phase speed c a priori. The equations combine to yield:

$$\frac{\partial^2 \Phi'}{\partial t^2} - c^2 \frac{\partial^2 \Phi'}{\partial x^2} = \frac{\partial \mathcal{Q}}{\partial t}$$

Weak temperature gradient balance

The two equations can be combined to form the forced (inhomogeneous) wave equation

$$\frac{\partial^2 \Phi'}{\partial t^2} - c^2 \frac{\partial^2 \Phi'}{\partial x^2} = \frac{\partial \mathcal{Q}}{\partial t}$$

Where we can break down the heating into

$$\mathcal{Q} = F(x)H(t)$$

Where H is the Heaviside step function.

Weak temperature gradient balance

The forced wave equation has a solution in the form of a Green's function, which can be written as:

$$\Phi'(x,t) = \frac{1}{2c}\int_0^t \int_{x-ct}^{x+ct} F(x')\delta(t')dx'dt'$$

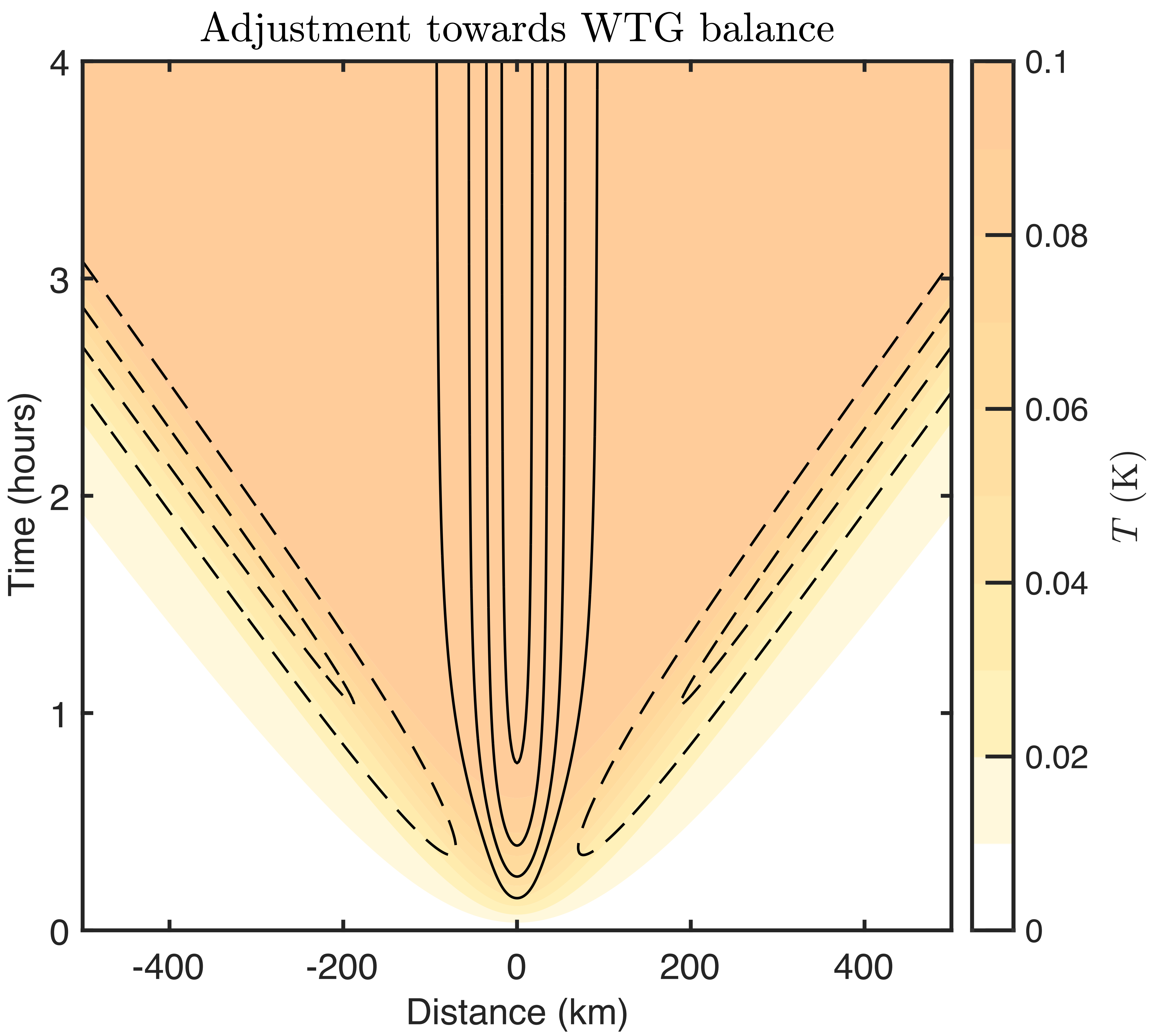
Where $\delta(t')$ is the Dirac delta function.

Note that x' and t' are different from x and t.

The forced wave equation

$$\Phi'(x,t) = \frac{1}{2c}\int_0^t \int_{x-ct}^{x+ct} F(x')\delta(t')dx'dt'$$

The solution shows gravity waves propagating away from the heat source, warming the column adiabatically as they propagate at the phase speed c.

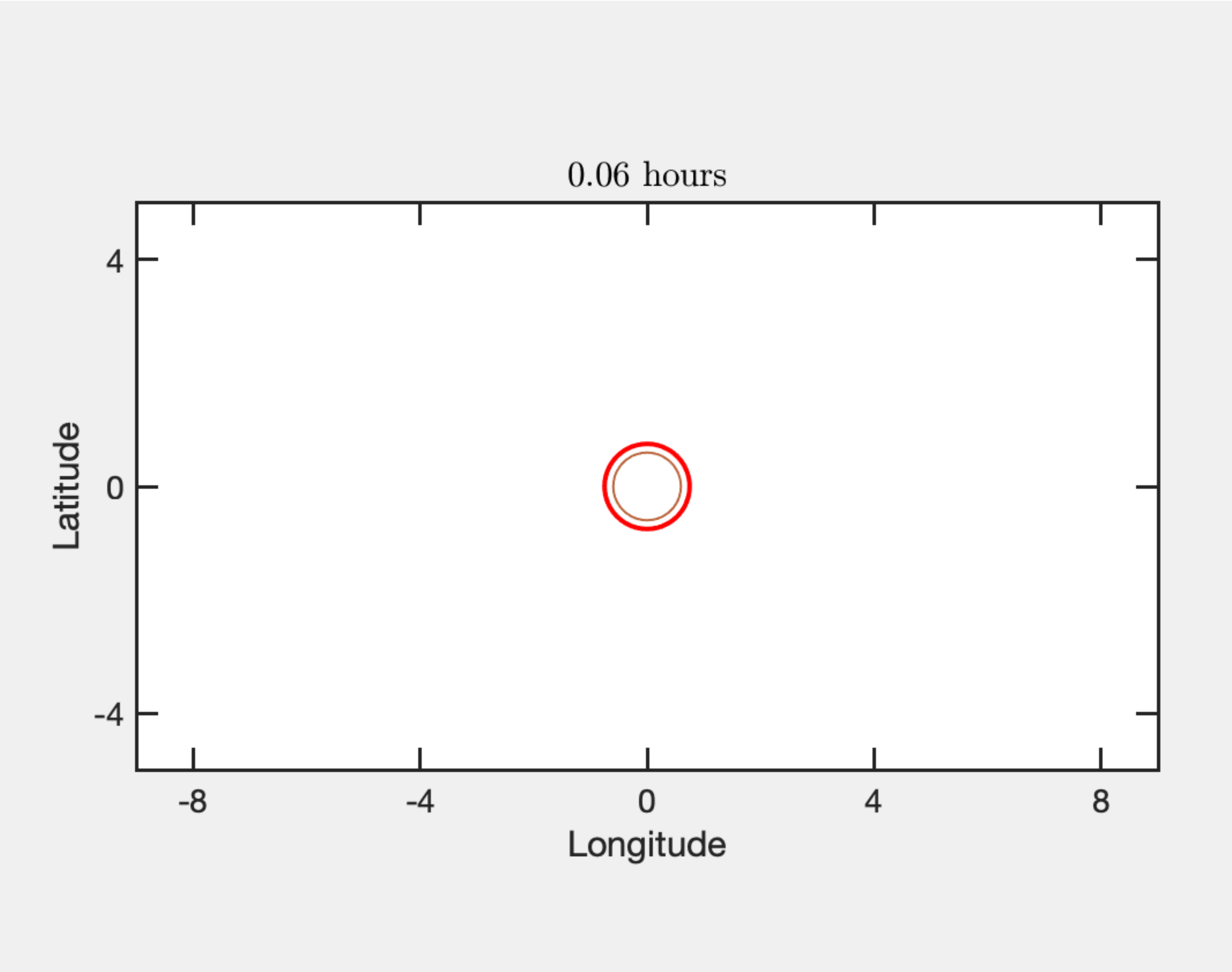


Contoured: vertical velocity (w)

2-D Forced wave equation

$$\frac{\partial^2 \Phi'}{\partial t^2} - c^2 \nabla_h \Phi' = \frac{\partial \mathcal{Q}}{\partial t}$$

The solution shows gravity waves propagating away from the heat source, warming the column as they propagate at the phase speed c.



Contoured: vertical velocity (w)

2-D Forced wave equation

This process is clearly seen in composites based on ERA5 data.

In this instant the gravity waves follow a phase speed of the first bar-clinic mode, which is roughly 50 m/s.