

Gravity waves:

$$\frac{\partial \vec{v}}{\partial t} = -\nabla_n \Phi \quad (1)$$

$$\frac{\partial}{\partial t} \left(\frac{\partial \Phi'}{\partial p} \right) = -\omega \sigma \quad \text{Therm (3)}$$

$$\nabla_n \cdot \vec{v} = -\frac{\partial \omega}{\partial p} \quad (2)$$

$$\frac{\partial \Phi'}{\partial p} = -\frac{R_d T'}{p}$$

$$\sigma = \frac{R_d \Delta p}{p C_p}$$

static
stability
param
like
 N^2 in
p-coord

Now can we combine these equations into one

Taking $\nabla_n \cdot (1)$ yields

$$\delta = \nabla_n \cdot \vec{v}$$

$$\frac{\partial \delta}{\partial t} = -\nabla_n^2 \Phi \quad \text{Div. Eqn.}$$

$$\delta = -\frac{\partial \omega}{\partial p}$$

$$-\frac{\partial^2 \omega}{\partial t \partial p} = -\nabla_n^2 \Phi \quad (4)$$

Let's take $\frac{\partial^2}{\partial t \partial p}$ of Eq. (3)

$$\frac{\partial^2}{\partial p \partial t} \frac{\partial}{\partial t} \left(\frac{\partial \Phi'}{\partial p} \right) = -\sigma \frac{\partial^2 \omega}{\partial p \partial t} = \sigma \nabla_n^2 \Phi$$

$$\frac{\partial^4 \Phi'}{\partial p^2 \partial t^2} = -\sigma \nabla_n^2 \Phi' \quad \text{"savage!" - HC}$$

Let's assume σ is a constant.

$$\Phi = \hat{\Phi} \exp(ikx + i\ell y + imp - i\omega t)$$

$$\frac{\partial^2 \hat{\Phi}}{\partial p^2} = -m^2 \hat{\Phi} \quad (\text{recall isentropic balance class})$$

$$(-m^2)(-\omega^2) \hat{\Phi} = -\sigma (-k^2 - \ell^2) \hat{\Phi} \quad K^2 = k^2 + \ell^2$$

$$m^2 \omega^2 = \sigma K^2$$

$$\omega = \frac{K \sqrt{\sigma}}{m}$$

$$c = \frac{\omega}{k} = \frac{\sqrt{\sigma}}{m} = \frac{\sigma}{m s^{-1}}$$

gravity wave phase
speed