AOS 801: Advanced Tropical Meteorology

Lecture 9 Spring 2023

Vertical Velocity, Precipitation, Buoyancy and Water Vapor

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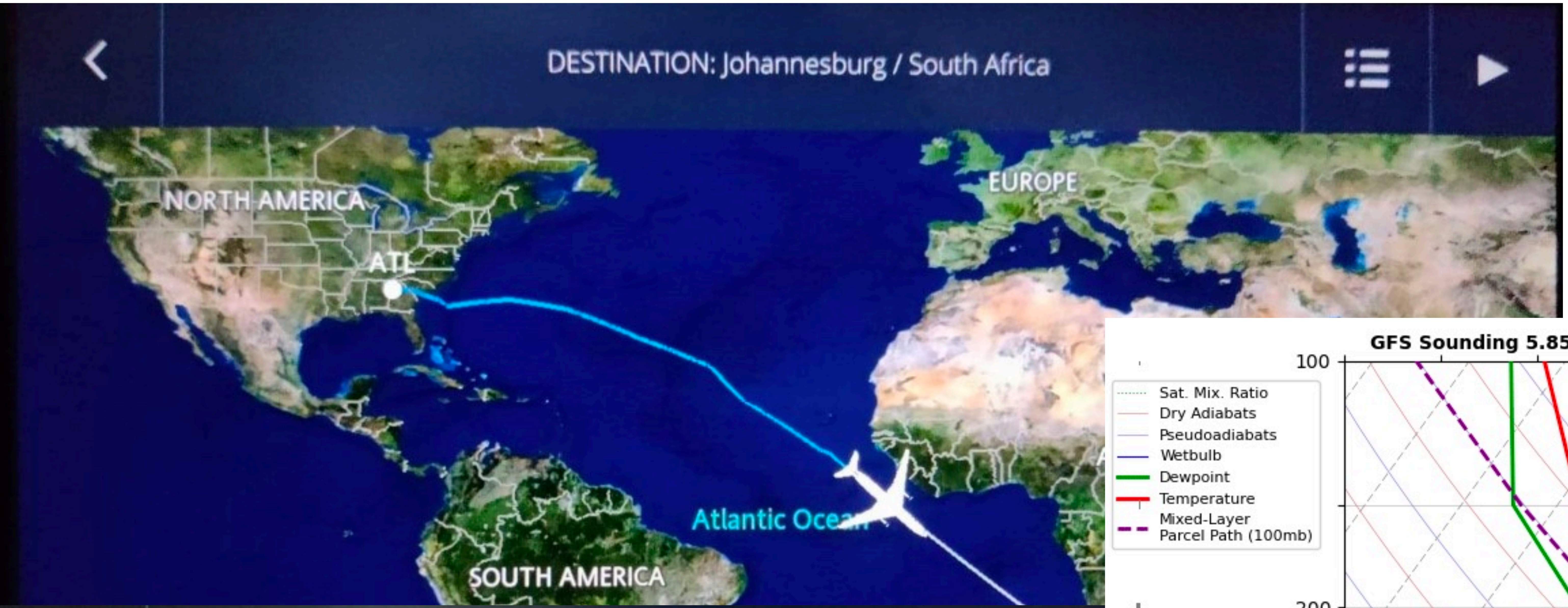
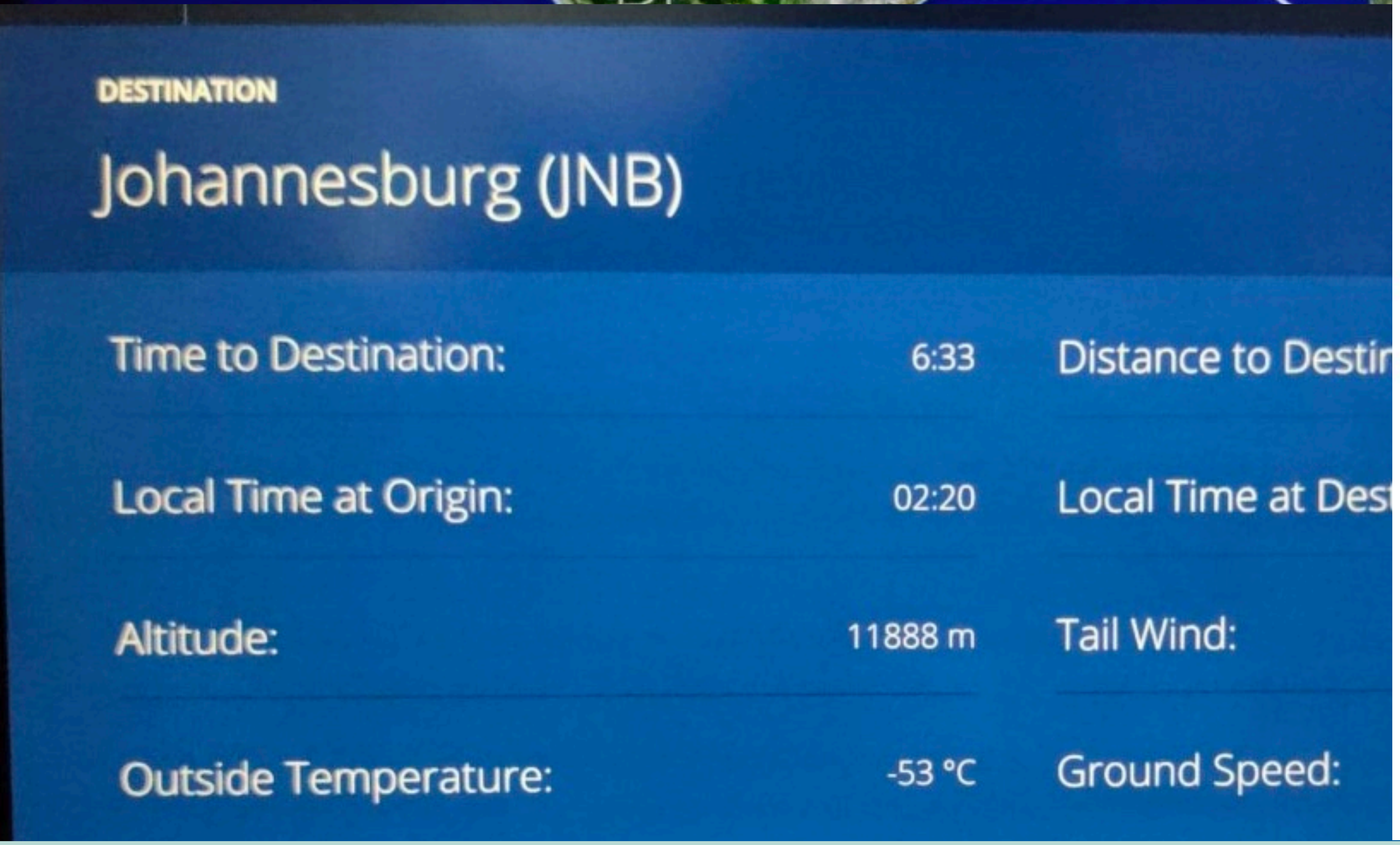
Announcements

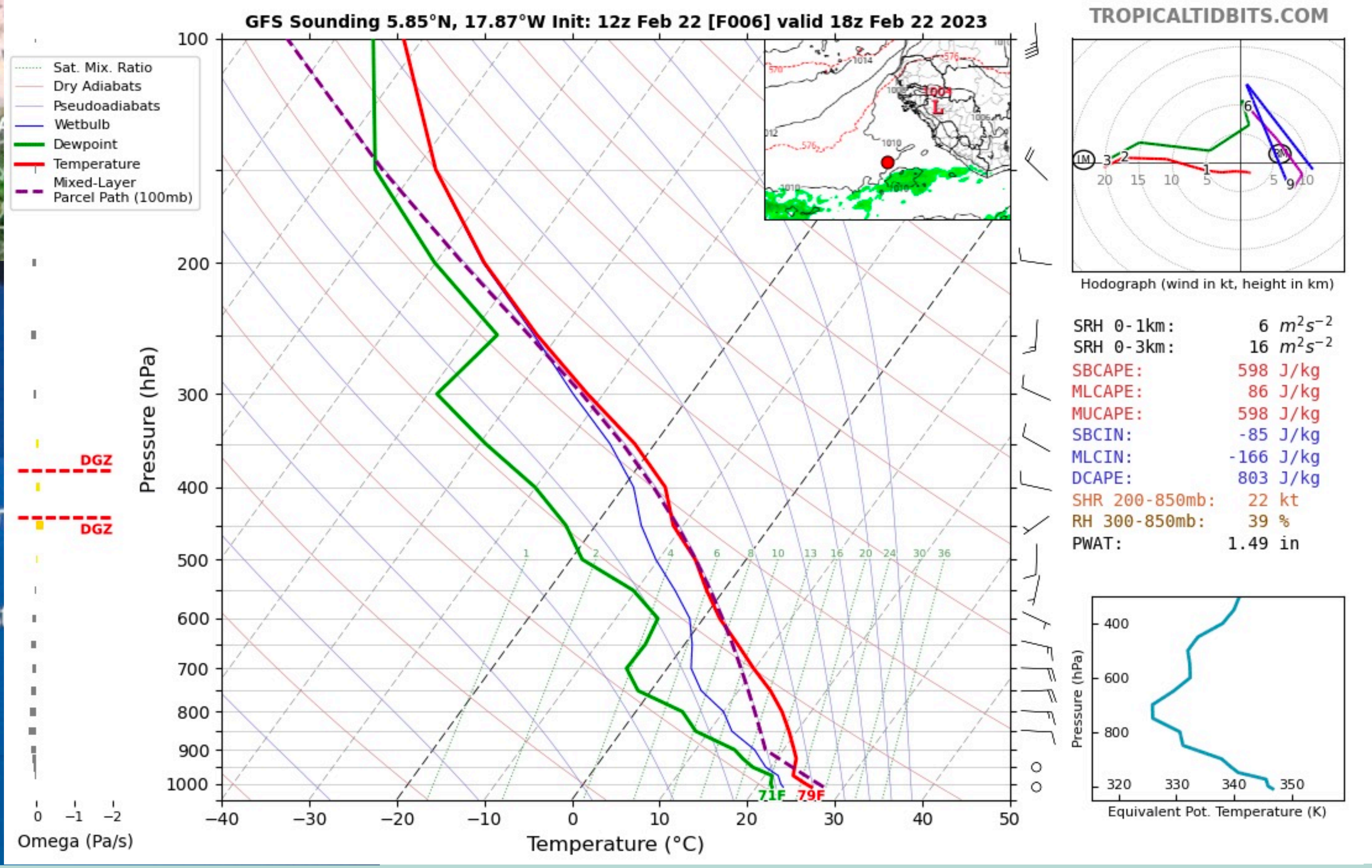
Remember the paper discussion on Monday.

HW 2 is due March 6.

Where in the world is Ian Beckley







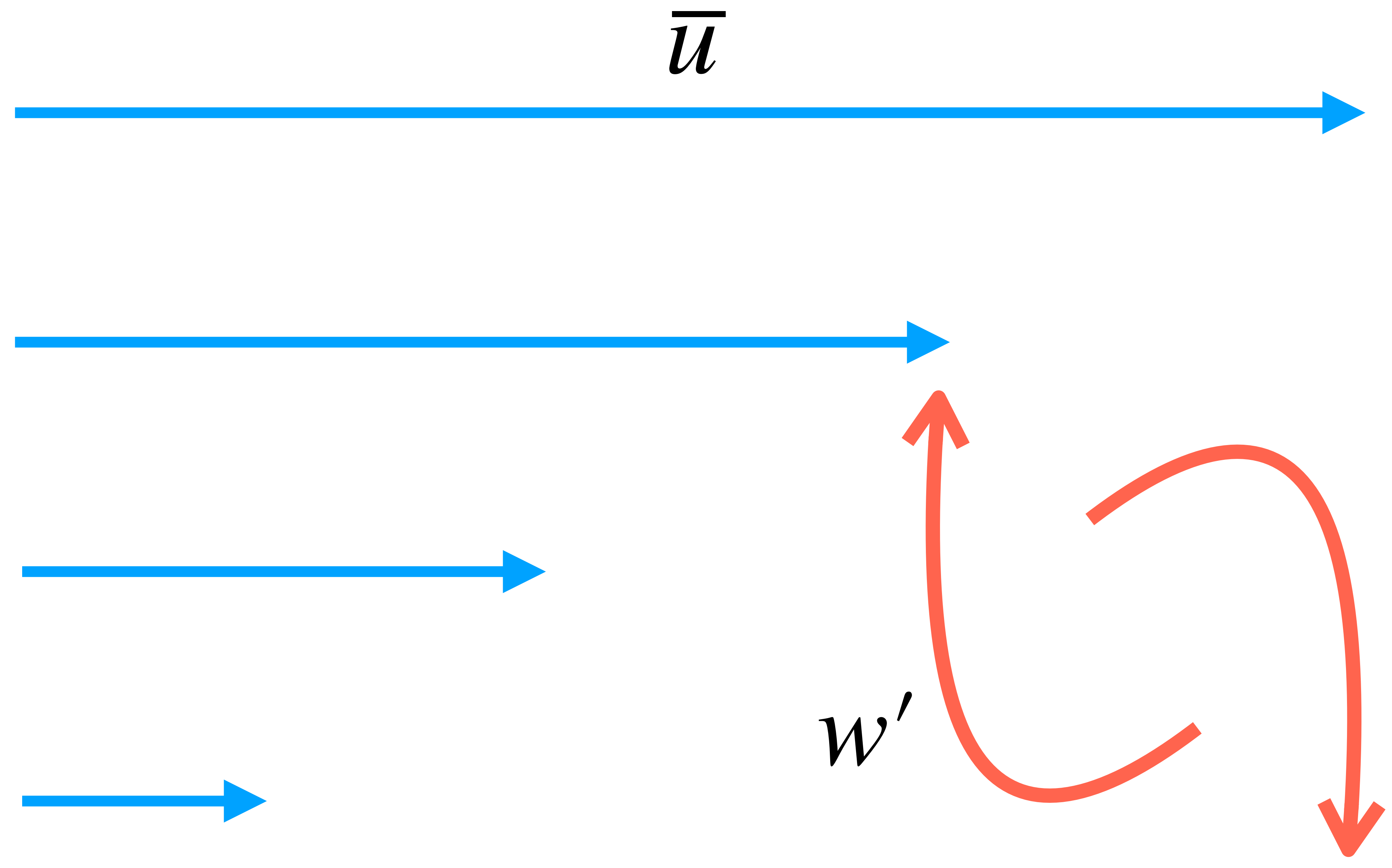
Flux gradient theory

Commonly used for boundary layer processes, the theory states that

$$\bar{\rho}\overline{u'w'} = -\mu_c \frac{\partial \bar{u}}{\partial z}$$

Eddy momentum fluxes are downgrading and dampening.

Turbulent entrainment of momentum acts as a diffusion.



Mixing length hypothesis

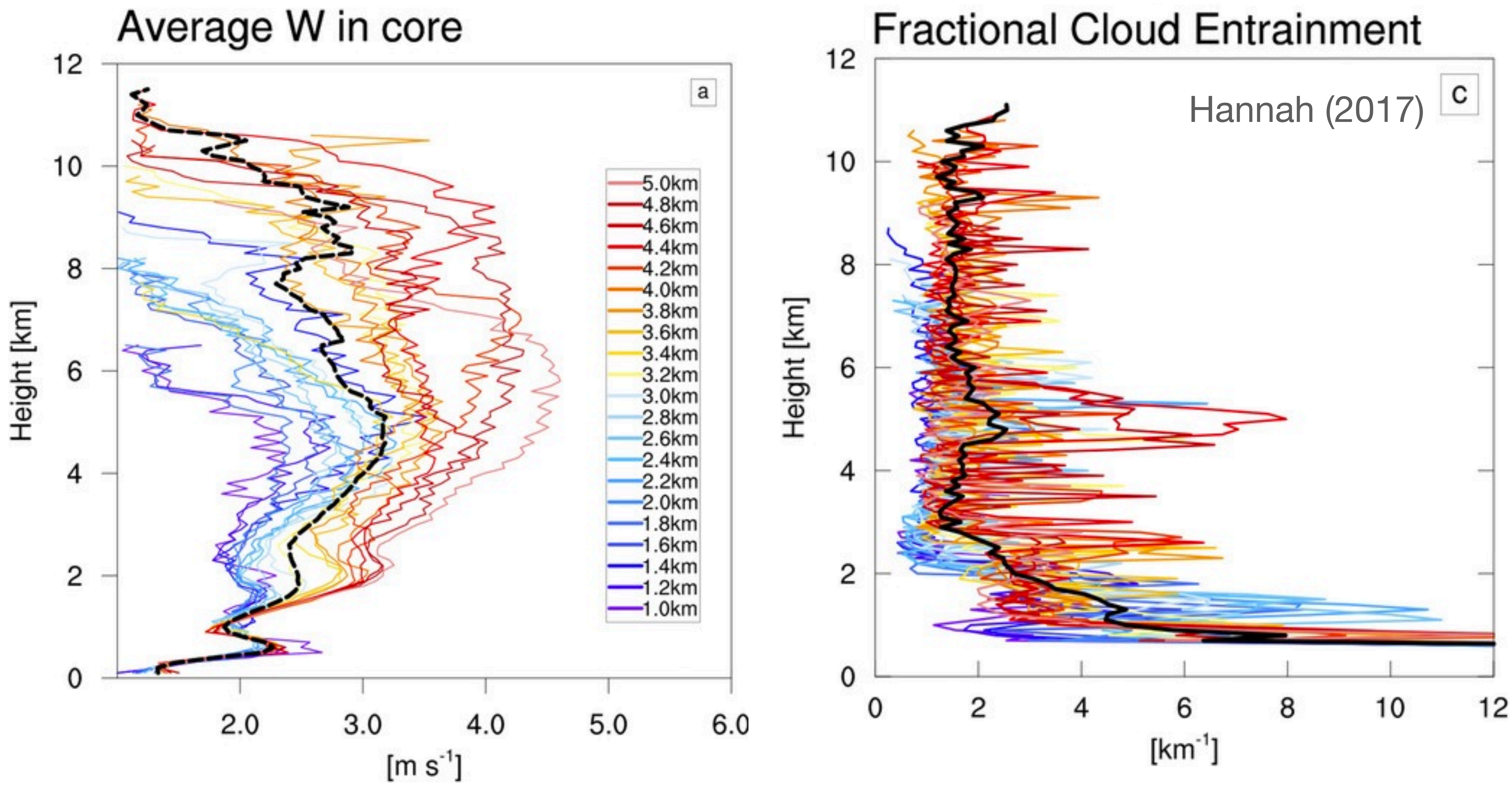
The eddy exchange coefficient (aka the eddy diffusivity) can be interpreted as the product of a mixing length and a mass flux scale

$$\mu_c = \rho U \ell$$

For convection, the mixing length is the inverse of the entrainment and the convective mass flux ($M_c = \rho \sigma w_c$, where σ is the fractional area covered by convection):

$$\mu_c = \frac{M_c}{\epsilon}$$

Average properties of convection



Assuming the whole domain is filled with these updrafts and downdrafts, we get

$$\mu_c \sim \frac{\rho W_c}{\epsilon} = \frac{1}{10^{-3}} = 10^3 \; kg/(ms)$$

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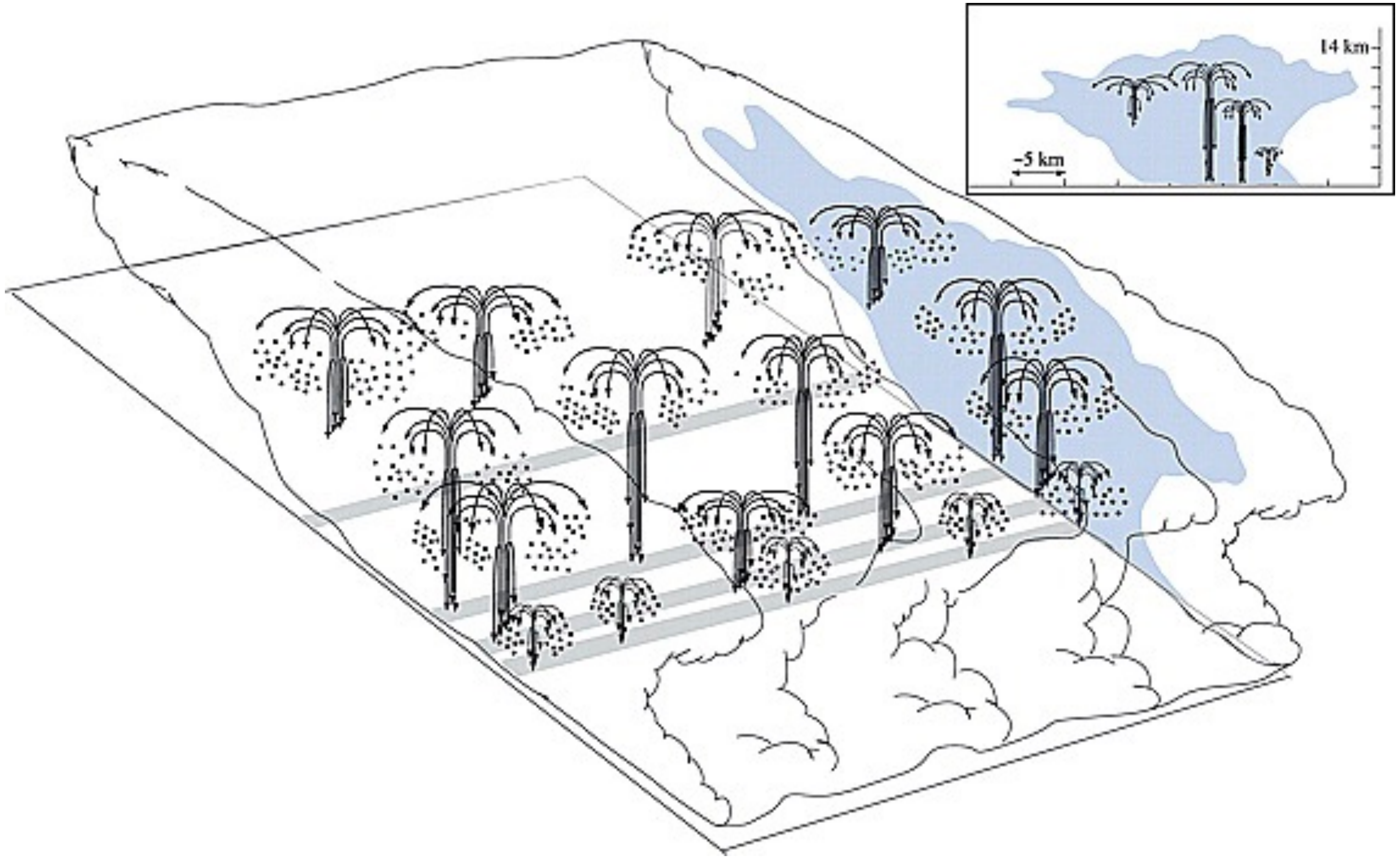

Houze (2004)

System of equations

Our basic equations with momentum diffusion are:

$$\frac{D\overline{\mathbf{v}}}{Dt} = -\frac{1}{\overline{\rho}}\nabla_h \overline{p}' - f\mathbf{k}\times\overline{\mathbf{v}} + \frac{\mu_c}{\overline{\rho}}\frac{\partial^2\overline{\mathbf{v}}}{\partial z^2}$$

$$\frac{D\overline{w}}{Dt} = -\frac{1}{\overline{\rho}}\frac{\partial\overline{p}'}{\partial z} + \overline{B} + \frac{\mu_c}{\overline{\rho}}\frac{\partial^2\overline{w}}{\partial z^2}$$

Non-hydrostatic pressure gradient force

Buoyancy

Coriolis force

Eddy momentum diffusion

Can we simplify these?

Scale Analysis

Two leading balances show up when the "**convective Reynolds number**" is small

$$\mathrm{Re}_c \equiv \frac{\rho \mathrm{WH}}{\mu_c} \qquad\qquad \mathrm{Re}_c \ll 1$$

What does this number mean?

$$\mathrm{Re}_c \equiv \frac{\rho \mathrm{WH}}{\rho W_c \ell}$$

The rate of mixing by turbulent updrafts and downdrafts must be much larger than the rate of mixing of the larger MCS.

Scale Analysis

Two leading balances show up when the "**convective Reynolds number**" is small

$$\mathrm{Re}_c \equiv \frac{\rho \mathrm{WH}}{\mu_c} \qquad \qquad \mathrm{Re}_c \ll 1$$

When $\mu_c = 10^3$ $\quad W = 10^{-2}\ m\ s^{-1}$ $\quad H = 10^4\ m$

Not unreasonable numbers

Leading balances are:

$$\nabla_h \overline{p}' \simeq \mu_c \frac{\partial^2 \overline{\mathbf{v}}}{\partial z^2} \qquad \qquad \frac{1}{\overline{\rho}} \frac{\partial \overline{p}'}{\partial z} \simeq \overline{B}\,.$$

Antitriptic Balance **Hydrostatic Balance**

Leading Balance in Convection

We can use mass continuity to merge the two equations to obtain the following:

$$\frac{\partial^3}{\partial z^3}\left(\frac{1}{\rho}\frac{\partial \rho w}{\partial z}\right) \simeq -\frac{\rho}{\mu_c}\nabla_h^2 B$$

In pressure coordinates you get an even simpler relation:

$$\frac{\partial^4 \omega}{\partial p^4} = \frac{1}{\mu_c^*}\nabla_h^2 \alpha'$$

$$\alpha' = R_d T'/p \qquad T' = T_c - T_0 \qquad \mu_c^* = \omega_c p_\ell \sim g^2 \mu_c \sim 10^5$$

What does this mean?