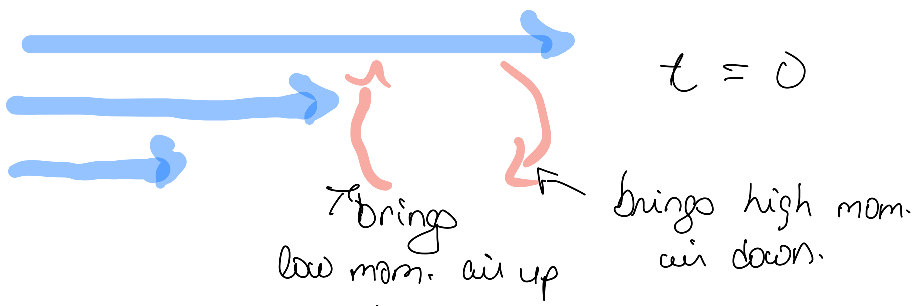
Turbulent Mixing in deep convection

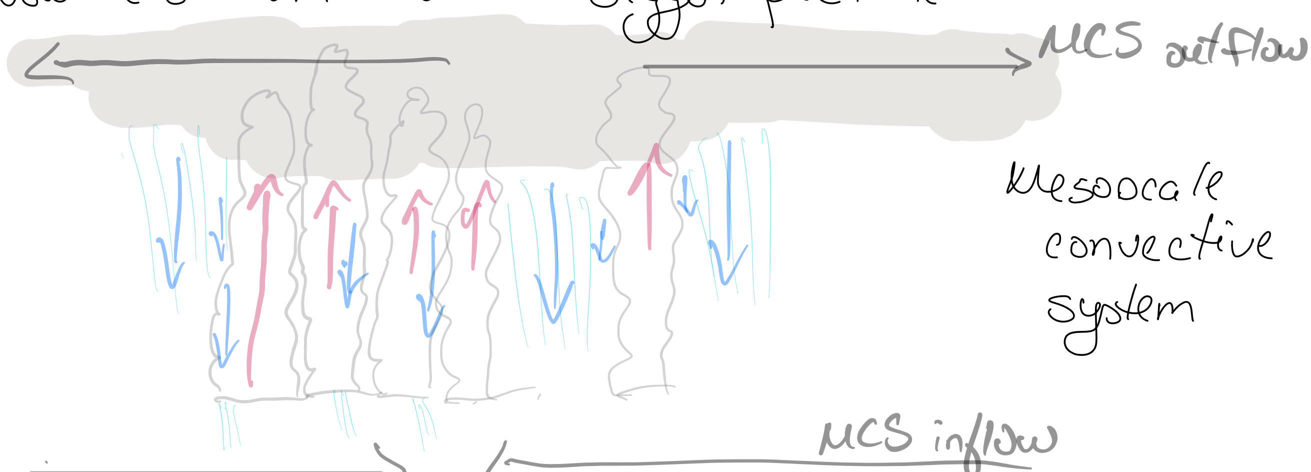
u' = high frequency eddy smaller than domain $\bar{u}$ = domain-mean winds

In BL meteorology, turbulence acts "downgradient", mixing winds in the BL, therefore reducing the gradient.



In general, turbulent eddies mix the air, and homogenize the winds:

Now let's think about a bigger picture



The updrafts and downdrafts would weaken the MCS
mesoscale winds

This qualitative picture implies that convective updrafts and downdrafts can act as a diffusion so that our mom. eqns. can be written as:

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho}\nabla_h p' - f\hat{k}\times\vec{v} + \frac{\mu_c}{\rho}\frac{\partial^2\vec{v}}{\partial z^2}$$

$$\frac{D\bar{w}}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + \bar{B} + \frac{\mu_c}{\rho}\frac{\partial^2 w}{\partial z^2}$$

Can we simplify?

Let's do scale analysis:

$$\underbrace{(x, y)}_{\text{dimensional var}} = \underbrace{L}_{\text{length scale}}\underbrace{(\hat{x}, \hat{y})}_{\text{non dim. var}} \qquad z = H\hat{z} \qquad t = \frac{L}{U}\hat{t}$$

$$(\bar{u}, \bar{v}) = U(\hat{u}, \hat{v}) \qquad \bar{w} = W\hat{w}$$

Let's assume that p' is part of the leading-order balance

$$\frac{U^2}{L}\frac{D\hat{\vec{v}}}{D\hat{t}} = -X\hat{\nabla}_h\hat{p}' - fU\hat{f}\hat{k}\times\hat{v} - \frac{\mu_c U}{\rho H^2}\frac{\partial^2\hat{v}}{\partial\hat{z}^2}$$

In GFD classes, we usually non-dim this eqn. by dividing by fU, and that gives you the Rossby number.

However, for the case of an MCS we divide by the last term instead.

$$\cancel{Re_c}\frac{D\hat{v}}{D\hat{t}} = -\hat{\nabla}_h\hat{p}' - \frac{Re_c}{Ro}\cancel{\hat{k}}\times\hat{v} + \frac{\partial^2\hat{v}}{\partial z^2} \qquad \text{Hor. Mom.}$$

$$p' = \frac{\mu_c L U}{H^2}\hat{p}' \qquad Ro = \frac{U}{fL} \quad \text{Rossby number}$$

$$Re_c = \frac{\rho W H}{\mu_c}$$ Convective Reynolds number

By using the same method:

$$Re_c \frac{H^2}{L^2} \frac{D\hat{w}}{D\hat{t}} = \frac{-\partial \hat{p}'}{\partial \hat{z}} + \hat{B} + \frac{H}{L} \frac{\partial^2 \hat{w}}{\partial \hat{z}^2}$$ Vert. Mom.

$$Re_c = \frac{\rho W H}{\mu_c} = \frac{10^0 10^{-2} 10^4}{10^3} = 0.1$$

Meaning that turbulent convective mixing ≫ mesoscale mixing

Now we have antitriptic balance in hor. mom. and hydrostatic balance in vert. mom.
What if we combine the equations.