

AOS 801: Advanced Tropical Meteorology  
*Lecture 8 Spring 2023*  
The Nature of Tropical Deep Convection

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# Announcements

Please turn in HW1 and PA1 by the end of the day today.

HW2 and PA2 should be out by the end of the day. Due in Two weeks

We will discuss paper 2 on next Monday's class. Wheel of fortune discussion leaders.

# The entraining plume hypothesis

Integrating from the top of the boundary layer we find

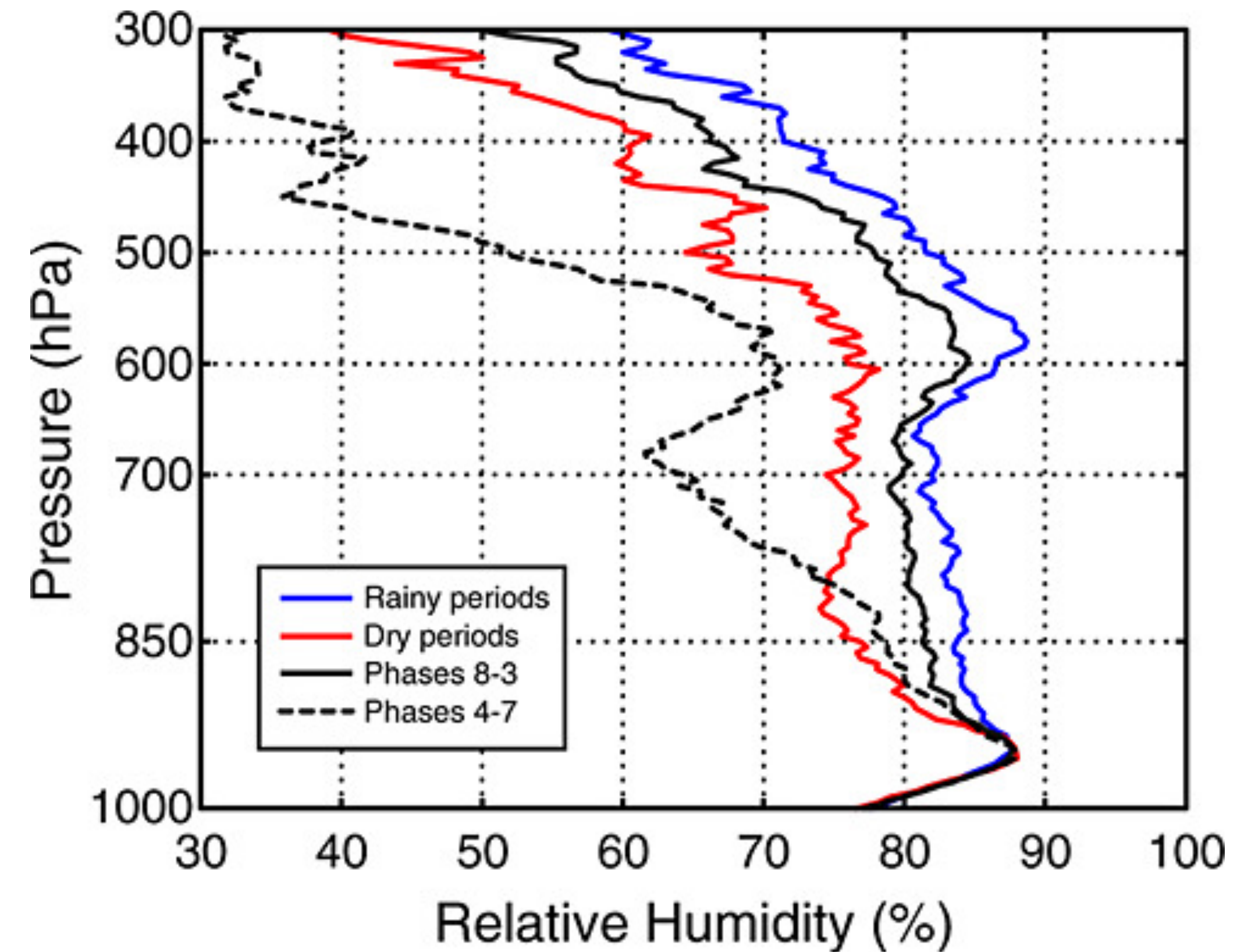
$$B = \underbrace{g \frac{\text{MSE}_B - \text{MSE}_e^*}{\kappa C_p T_e}}_{\text{Undilute component (B}_U\text{)}} - \underbrace{\frac{g}{\kappa C_p T_e} \int_{z_B}^z \epsilon L_v q^* (1 - \text{RH}) dz'}_{\text{Dilution (D}_B\text{)}}$$

Undilute component ( $B_U$ )

Dilution ( $D_B$ )

So we can write the buoyancy as

$$B = B_U - D_B$$



Powell and Houze (2013)

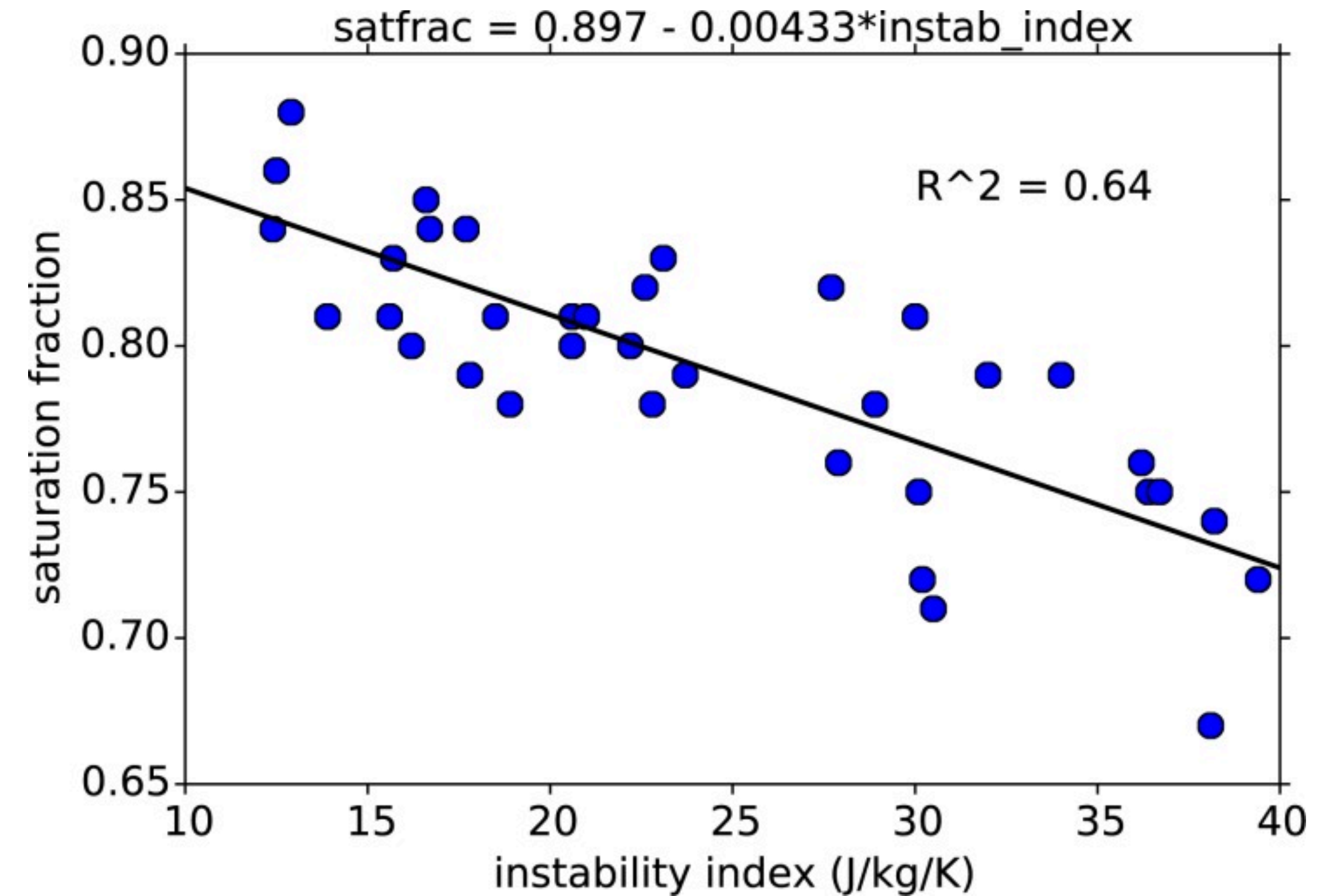
# Dilution and Convective Quasi-equilibrium

If we still assume that  $B \sim 0$  in the tropics overall we find that

$$B_U = D_B$$

**Dry regions are unstable.**

**Humid regions are stable.**

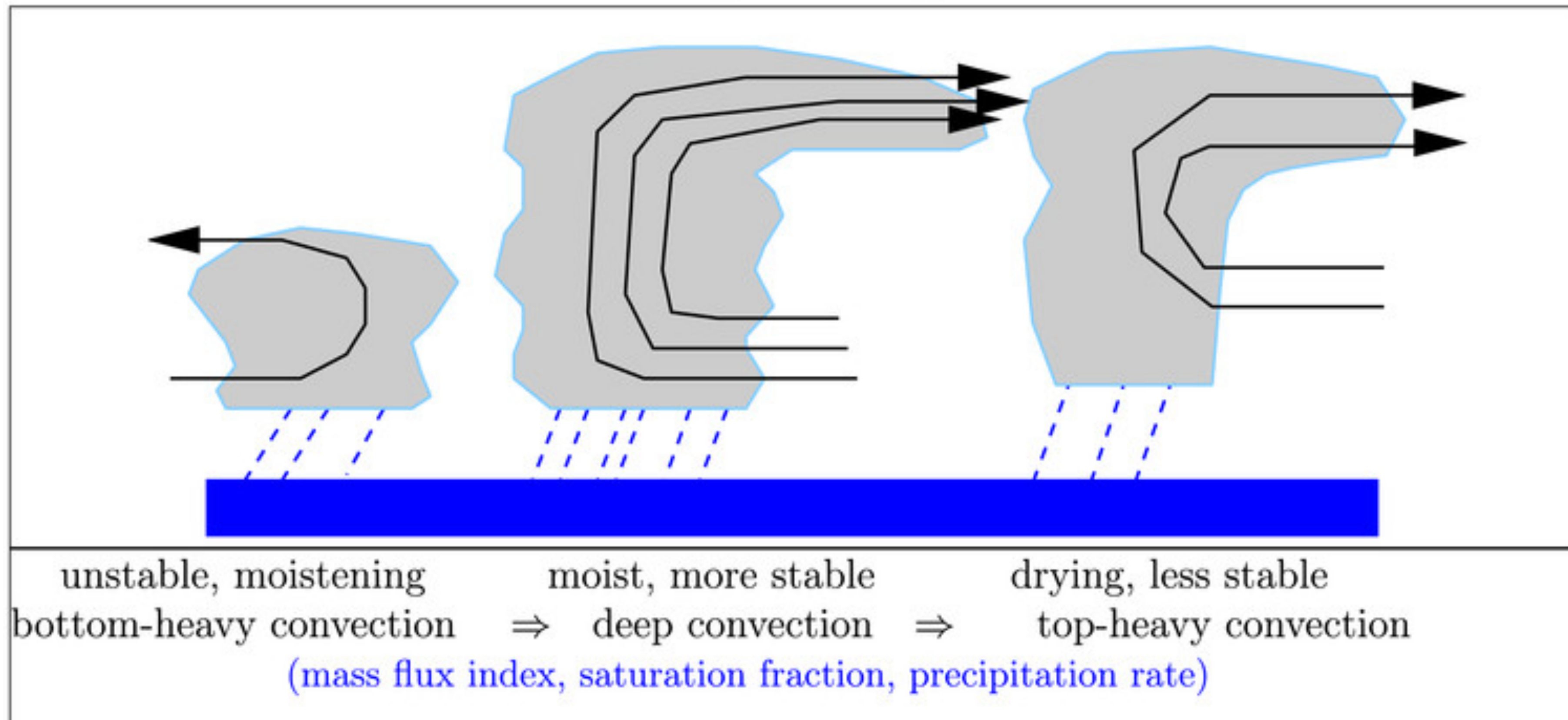


# Moisture Quasi-equilibrium

**Dry regions are unstable.**  
**Humid regions are stable.**

When dilution is considered **Convective QE**  
becomes **moisture QE**

moisture quasi-equilibrium



# The entraining moist adiabatic lapse rate

For an entraining plume, the lapse rate can be obtained from the MSE budget, yielding

$$\Gamma_{\epsilon m} = \left( \Gamma_d + \frac{\epsilon L_v q^+}{C_p} \right) \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}}. \quad q^+ = q^* - q$$

But this is just a moist adiabat with an extra term due to entrainment, so we write

$$\Gamma_{\epsilon m} = \Gamma_m + \Gamma_\epsilon. \quad \Gamma_m + \Gamma_\epsilon > \Gamma_m$$

The entraining moist adiabat is smaller than the moist adiabat, but by how much?

# The entraining moist adiabatic lapse rate

The entraining moist adiabat is smaller than the moist adiabat, but by how much? Let's do a quick examination

$$\Gamma_{em} = \left( \Gamma_d + \frac{\epsilon L_v q^+}{C_p} \right) \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}}.$$

The only terms that matter for the comparison are those inside the parenthesis

$$g + \epsilon L_v q^+ = ?$$

In general, entrainment leads to a modest correction to the moist adiabat in a humid environment ( $g \gg \epsilon L_v q^+$ ), but can be large in a dry environment ( $g \sim \epsilon L_v q^+$ )

# Moisture Quasi-Equilibrium: Intuitive form

Under quasi-equilibrium we have that

$$\Gamma_{em} = \Gamma_e$$

Implying that the environmental lapse rate  $\Gamma_e$  is steeper in a dry region than in a moist region. Thus, dry regions tend to be more unstable than moist regions, although moisture counteracts this.

Thus, it is not CAPE that changes little with time, but a different measure of instability that includes dry air entrainment:

$$\frac{\partial \text{CAPE}_\epsilon}{\partial t} \simeq 0$$

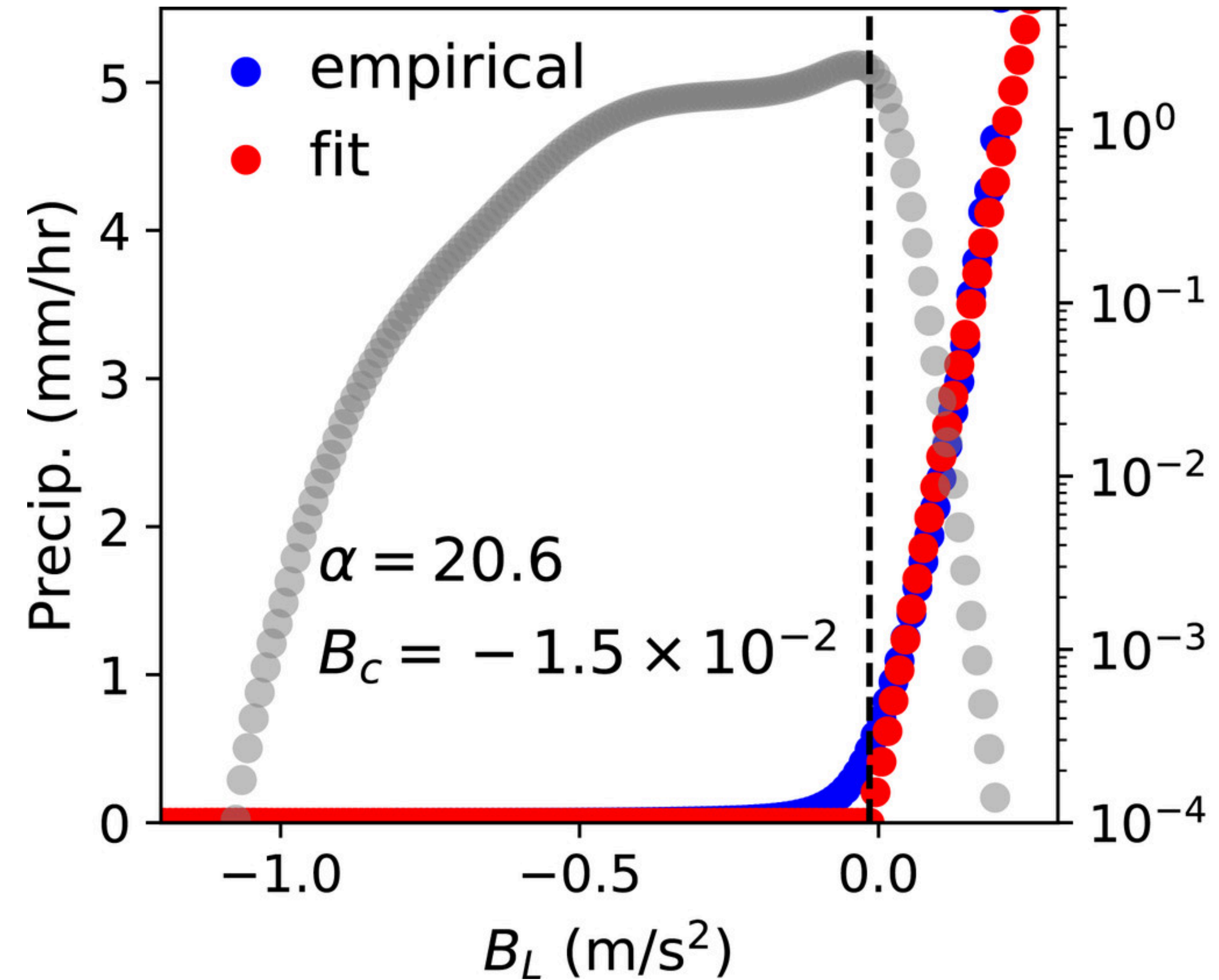
$\text{CAPE}_\epsilon$  is known as the entraining CAPE.



# Moisture Quasi-Equilibrium

Climatologically-speaking, areas of rainfall tend to hover around a near a fixed B value, consistent with MQE.

But the deviations from this stable point is what drives rapid changes in rainfall. Why?



Ahmed et al. (2020)

## **Discretion Warning:**

Much of the slides I'm presenting today are unpublished work done by my group and collaborators. Please do not share.

### ***Acknowledgement:***

Víctor C. Mayta, Kathleen Schiro, Brandon O. Wolding, Fiaz Ahmed, J. David Neelin, Larissa Back, and others.

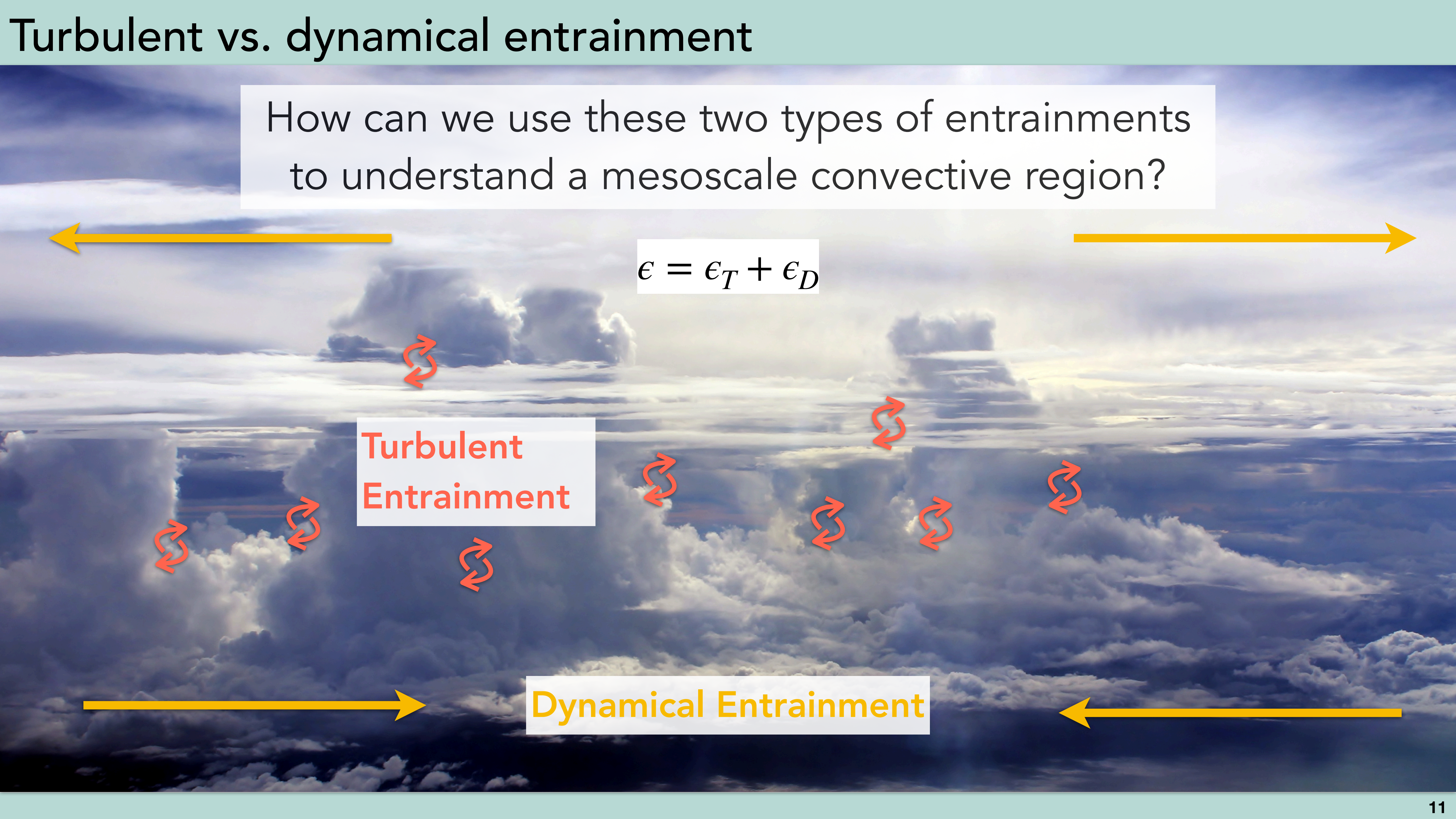
# Turbulent vs. dynamical entrainment

How can we use these two types of entrainments to understand a mesoscale convective region?

$$\epsilon = \epsilon_T + \epsilon_D$$

**Turbulent  
Entrainment**

**Dynamical Entrainment**



# Notation

Domain-mean variables have overlines



Convective elements within domain have subscript c

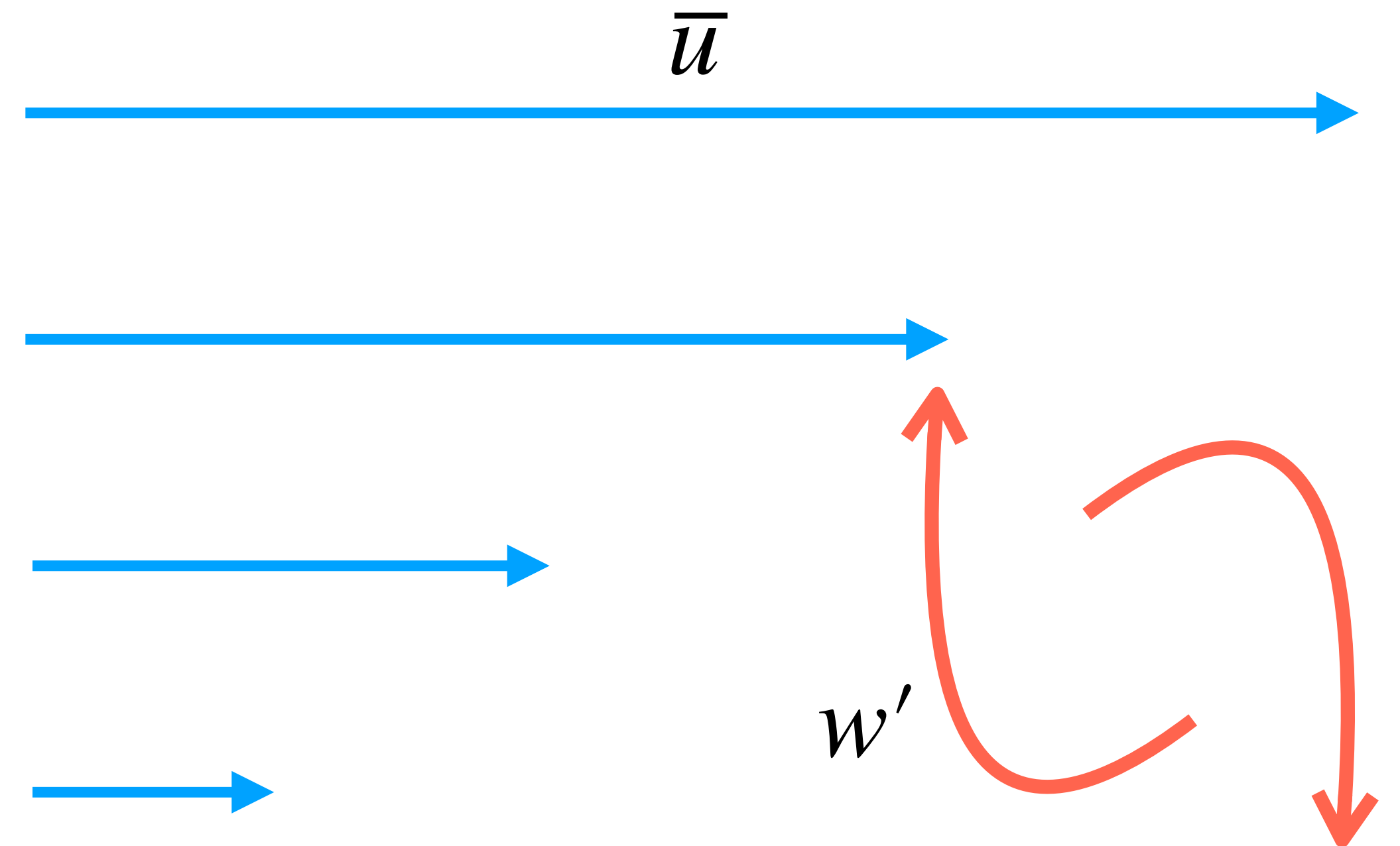
# Flux gradient theory

Commonly used for boundary layer processes, the theory states that

$$\overline{\rho u'w'} = -\mu_c \frac{\partial \bar{u}}{\partial z}$$

Eddy momentum fluxes are downgrading and dampening.

**Turbulent entrainment of momentum acts as a diffusion.**



# System of equations

Our basic equations with momentum diffusion are:

$$\frac{\overline{D}\overline{\mathbf{v}}}{Dt} = -\frac{1}{\overline{\rho}} \nabla_h \overline{p}' - f \mathbf{k} \times \overline{\mathbf{v}} + \frac{\mu_c}{\overline{\rho}} \frac{\partial^2 \overline{\mathbf{v}}}{\partial z^2}$$

Non-hydrostatic pressure gradient force

Buoyancy

Coriolis force

Eddy momentum diffusion

$$\frac{\overline{D}\overline{w}}{Dt} = -\frac{1}{\overline{\rho}} \frac{\partial \overline{p}'}{\partial z} + \overline{B} + \frac{\mu_c}{\overline{\rho}} \frac{\partial^2 \overline{w}}{\partial z^2}$$

Can we simplify these?