

Convective Quasi-Equilibrium:

$$\frac{\partial \text{MSE}_c^*}{\partial z} = 0 \quad \longrightarrow \quad \frac{\partial T_c}{\partial z} = -\Gamma_m$$

$$\text{CAPE} = g \int_{\text{LFC}}^{\text{LNB}} \frac{T_c - T_e}{T_e} dz = 0$$

$\therefore T_c = T_e$  so that if we take  $\partial/\partial z$   
 $\frac{\partial T_c}{\partial z} = \frac{\partial T_e}{\partial z} = -\Gamma_m$  the same result we  
 get when we discussed  
 RCE

Why does convection determine the height of the tropopause?

$\rightarrow$  the atmosphere is in hydrostatic balance

$$\frac{\partial \Phi}{\partial p} = -\alpha = \frac{R_d T}{p}$$

$$\int_{p_t}^{p_s} \frac{\partial \Phi}{\partial p} dp = \int_{p_t}^{p_s} -\alpha dp$$

$$\cancel{\Phi_s} - \Phi_t = - \int_{p_t}^{p_s} \frac{R_d T}{p} dp =$$

$$\Phi = gz \Rightarrow \Phi_s \approx 0$$

if  $z_s = 0$

$$\Phi_t = - \int_{p_t}^{p_s} R_d T dp$$

$$\bar{z}_t = - \frac{R_d}{g} \int_{p_t}^{p_s} T_e dp$$

Recalling that  $T_e = T_c$ , the height of the tropopause is determined by the convection...

Consequence: Tropopause goes up higher in a more humid atmosphere where  $\Gamma_m$  is smaller...

$$\frac{\partial z_t}{\partial t} = -\frac{R_d}{g} \int_{p_t}^{p_s} \frac{\partial t_c}{\partial t} d \ln p$$

Maxwell's relation  $\left(\frac{\partial T}{\partial p}\right)_{S_m^*} = \left(\frac{d\alpha}{dS_m}\right)_p$

$$d\alpha = \left(\frac{\partial T}{\partial p}\right)_{S_m^*} dS_m$$

$$\begin{aligned} \frac{\partial z_t}{\partial t} &= \frac{1}{g} \int_{p_t}^{p_s} \frac{\partial \alpha}{\partial t} dp = \frac{1}{g} \int_{p_t}^{p_s} \frac{\partial S_m}{\partial t} \left(\frac{\partial T}{\partial p}\right)_{S_m} dp \\ &= \frac{1}{g} \frac{\partial S_{mB}}{\partial t} \int_{p_t}^{p_s} \left(\frac{\partial T}{\partial p}\right)_{S_m} dp \end{aligned}$$

After integration over  $p$  and over  $t$  we <sup>boundary layer</sup> <sub>average</sub> get:

$$\bar{z}_t = \frac{(T_s - T_t) \bar{S}_{mB}}{g}$$

$\bar{z}_t$  is determined by convection

Convection is determined by the energy of the BL (boundary layer)