AOS 801: Advanced Tropical Meteorology

Lecture 4   Spring 2023

Weak Temperature Gradient Balance and Radiative-Convective Equilibrium

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Photo courtesy of Rosa M. Vargas Martes

Last Class: Maxwell's Relation

We can obtain several relations using MSE or moist entropy that will prove useful in this class

$$Tds_m = d\mathrm{MSE} = C_p dT + d\Phi + L_v dq$$

If we keep pressure constant:

$$T(ds_m)_p = C_p dT + L_v dq = (d\varepsilon_m)_p$$

If moist entropy is kept constant

$$(d\varepsilon_m)_{s_m} = -\,\alpha(dp)_{s_m}$$

Cross differentiation yields

$$\left(\frac{dT}{dp}\right)_{s_m} = \left(\frac{d\alpha}{ds}\right)_p$$

The moist adiabatic lapse rate

A parcel that rises moist adiabatically conserves its MSE as it rises

$$\frac{D\mathrm{MSE}}{Dz} \approx 0$$

By expanding the definition and after some algebra and rearranging, we can obtain the moist adiabatic lapse rate

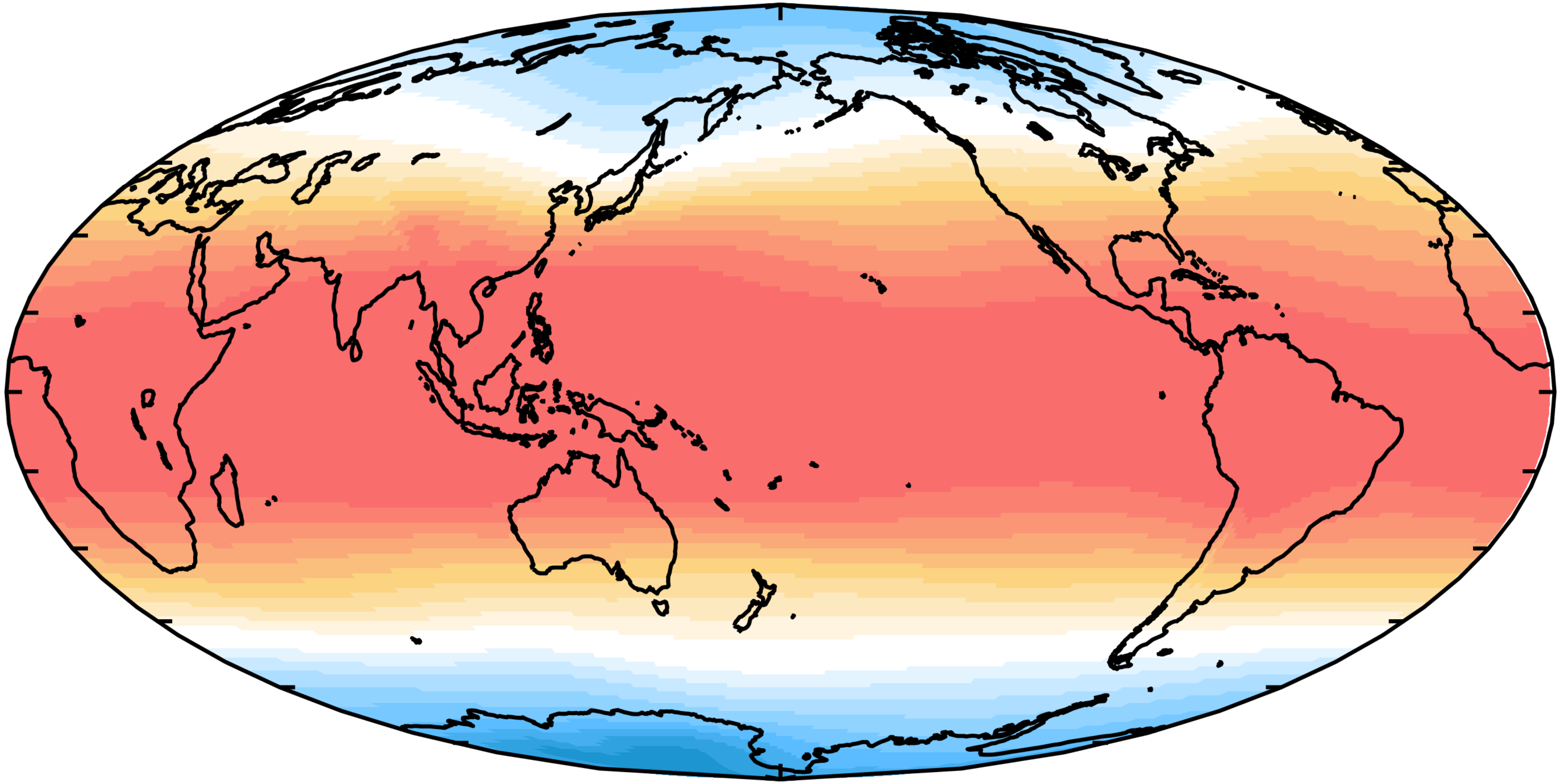
$$\frac{dT}{dz} = -\Gamma_m \qquad \Gamma_m = \Gamma_d \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}} \qquad \Gamma_d = \frac{g}{C_p}$$

Is the **moist adiabatic lapse rate.**

A gentle introduction to the weak temperature gradient approximation.

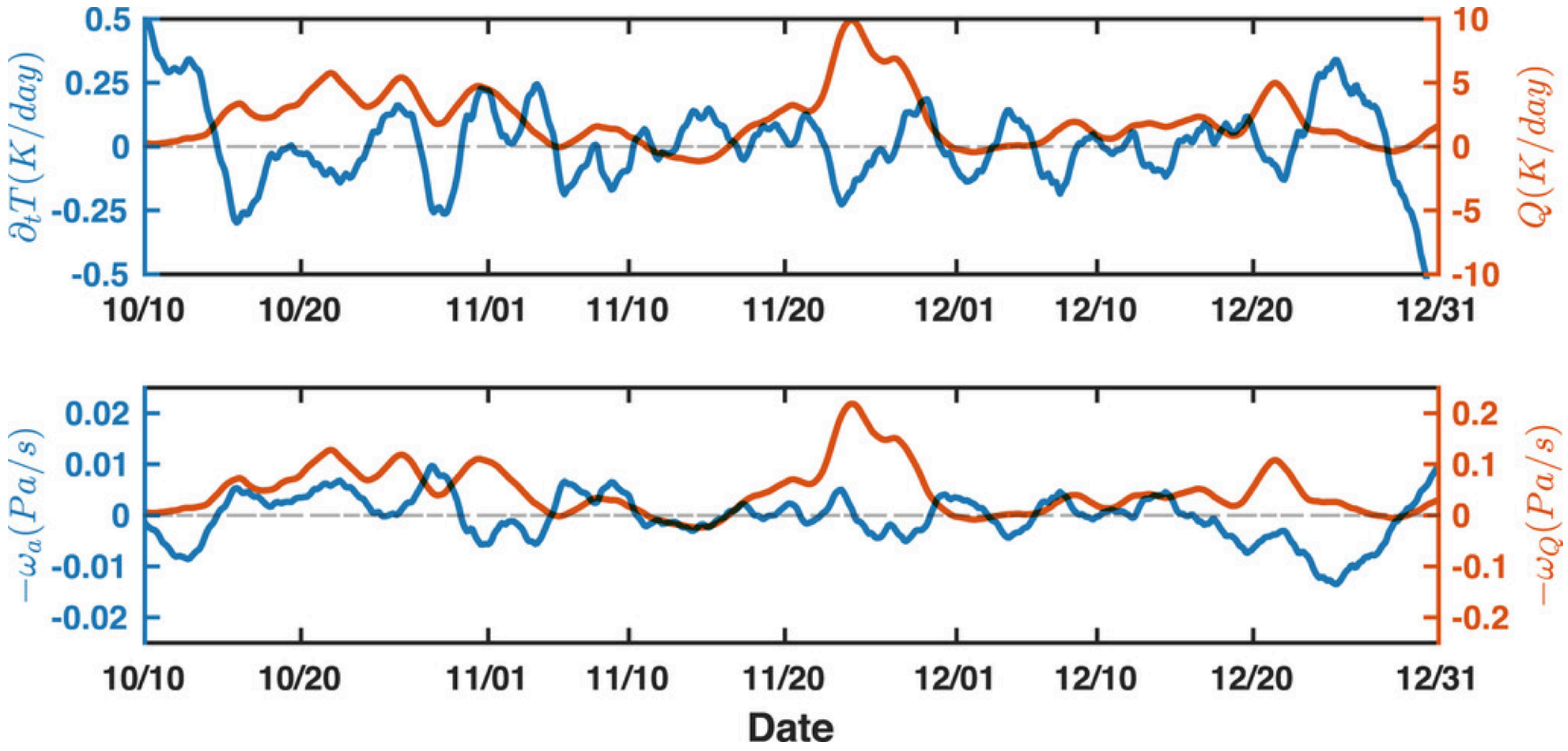
An introduction to the WTG approximation

$$C_p \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_h C_p T + \omega \frac{\partial \mathrm{DSE}}{\partial p} = Q \,.$$

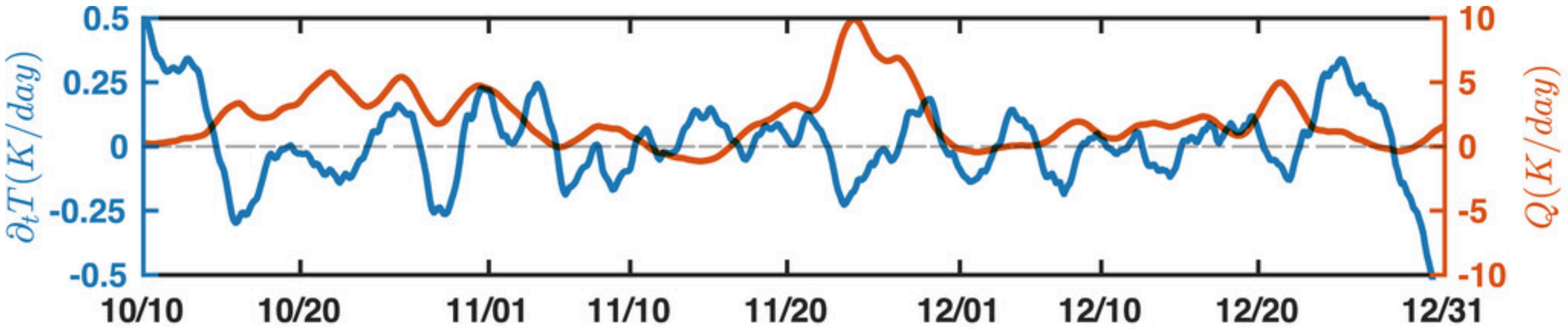


An introduction to the WTG approximation

The diabatic heating is actually much larger than the temperature tendency. Something must be balancing it.



The great deceit



Given the name "**diabatic heating**" there is a tacit assumption that the heating will cause a **warming**, i.e. a change in temperature. This is **not** true!

$$C_p \frac{\partial T}{\partial t} \neq Q \, .$$

The diabetic heating is instead in a thermodynamic balance with vertical DSE advection.

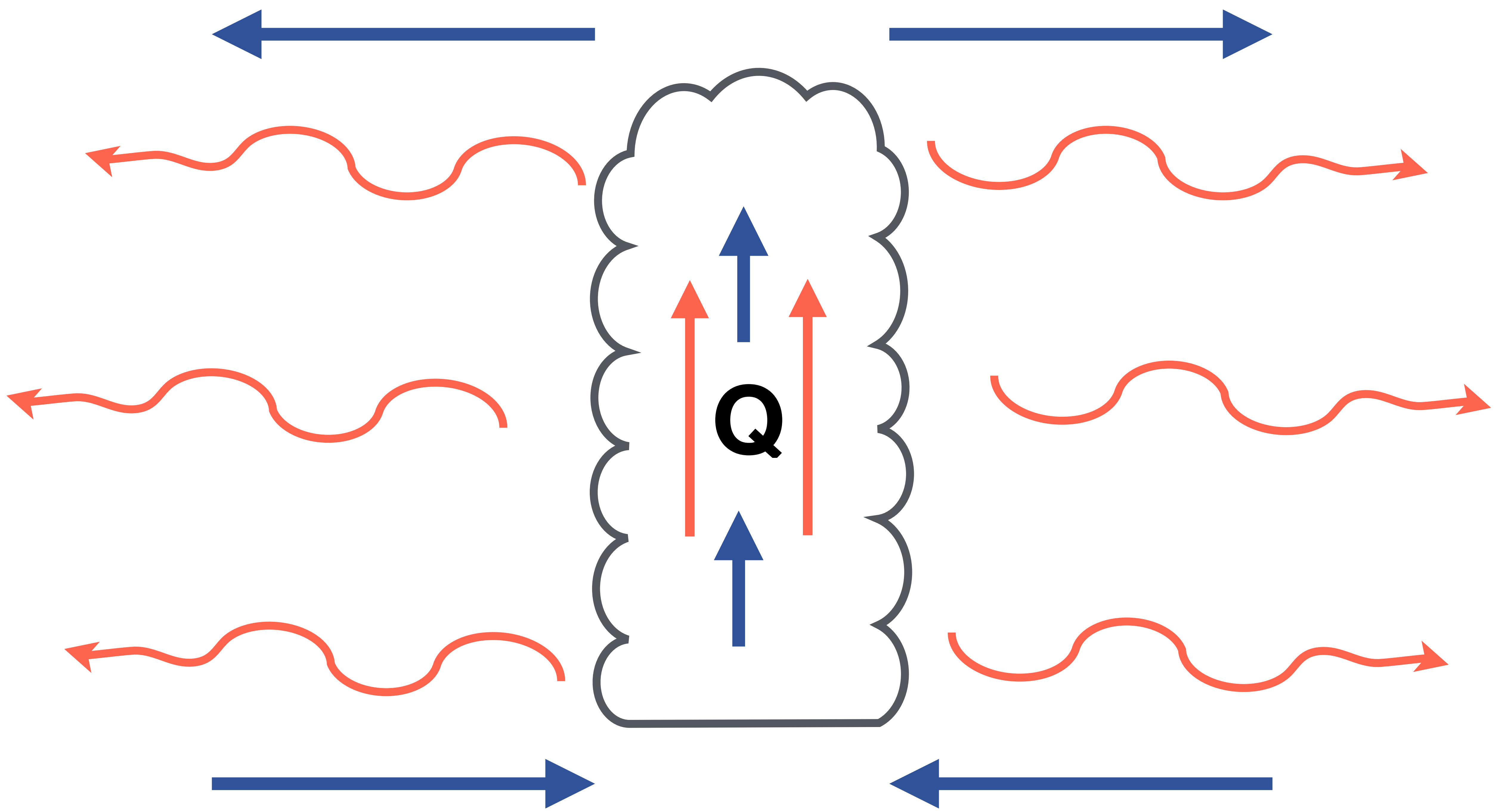
An introduction to the WTG approximation

$$C_p \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_h C_p T + \omega \frac{\partial \mathrm{DSE}}{\partial p} = Q \,.$$

Thus the leading order balance in the thermodynamic equation is:

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} \simeq Q$$

Adiabatic Cooling **Diabatic Heating**



Weak temperature gradient balance

Reynolds Averaging

Reynolds Averaging

Clouds are messy. In order to get around this problem we often invoke"Reynolds averaging" on a larger domain and represent the impact that convection has on the large-scale circulations

Clouds courtesy of Qiao-Jun Lin

Reynolds Averaging

What if we decompose the fields into domain-mean averages ($\overline{v}$) and fluctuations from this mean that occur within the domain (v')?

Clouds courtesy of Qiao-Jun Lin

Reynolds Averaging

With this averaging we obtain the following moisture and MSE

Thermodynamic budget

$$\frac{\partial C_p \overline{T}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla_h C_p \overline{T} + \omega \frac{\partial \overline{\mathrm{DSE}}}{\partial p} = Q_1 \,.$$

Apparent heating

$$Q_1 \equiv \overline{Q}_c + \overline{Q}_r - \frac{\partial \overline{\omega' \mathrm{DSE}'}}{\partial p}$$

Moisture budget

$$\frac{\partial L_v \overline{q}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla_h L_v \overline{q} + \overline{\omega} \frac{\partial L_v \overline{q}}{\partial p} = -\, Q_2 \,.$$

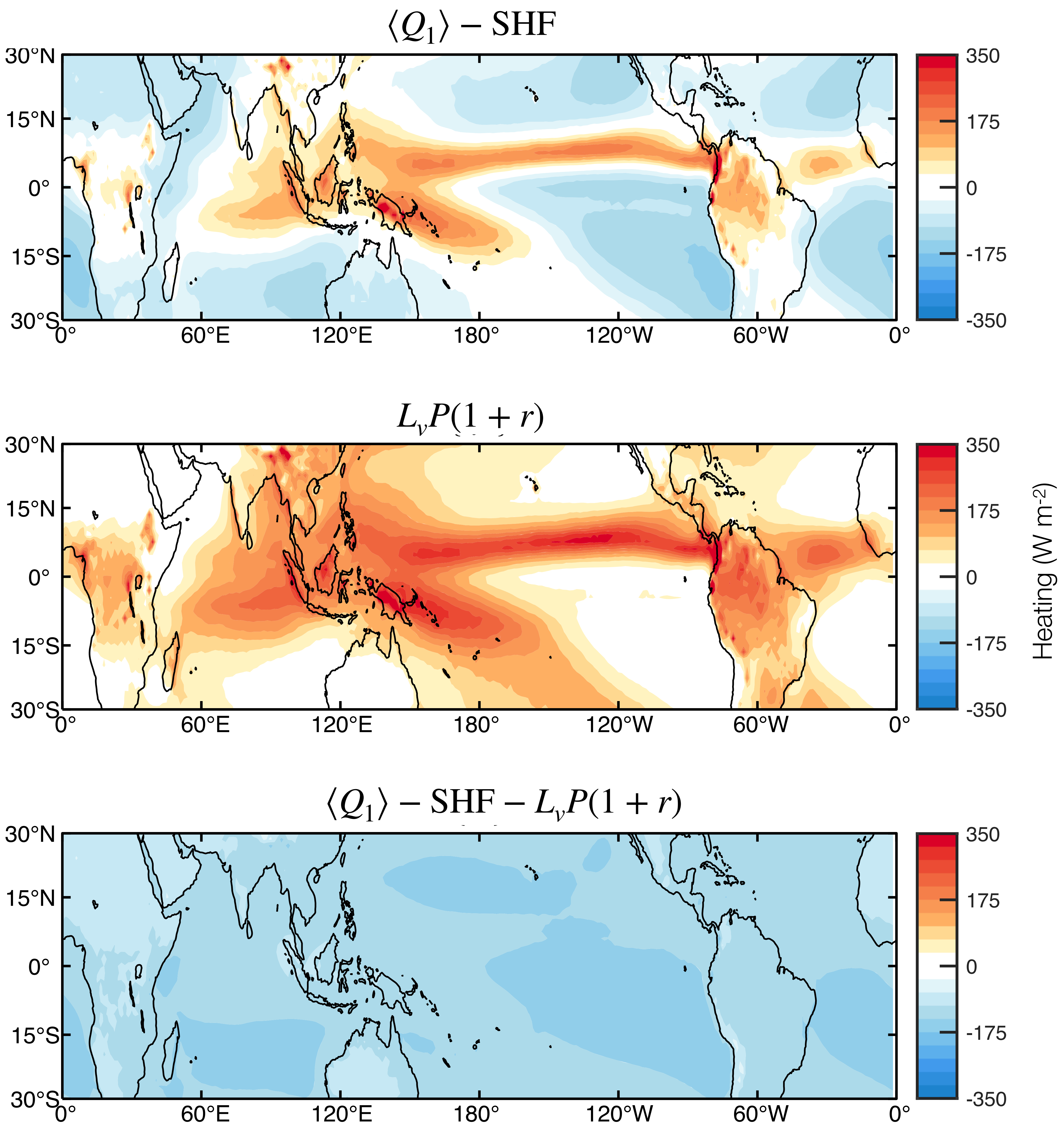
Apparent moisture sink

$$Q_2 \equiv -\, L_v \overline{S}_q + \frac{\partial \overline{\omega' L_v q'}}{\partial p}$$

From here on we will drop the overlines and assume that we are looking at large-scale averages.

Radiative-Convective equilibrium

Radiative Convective Equilibrium

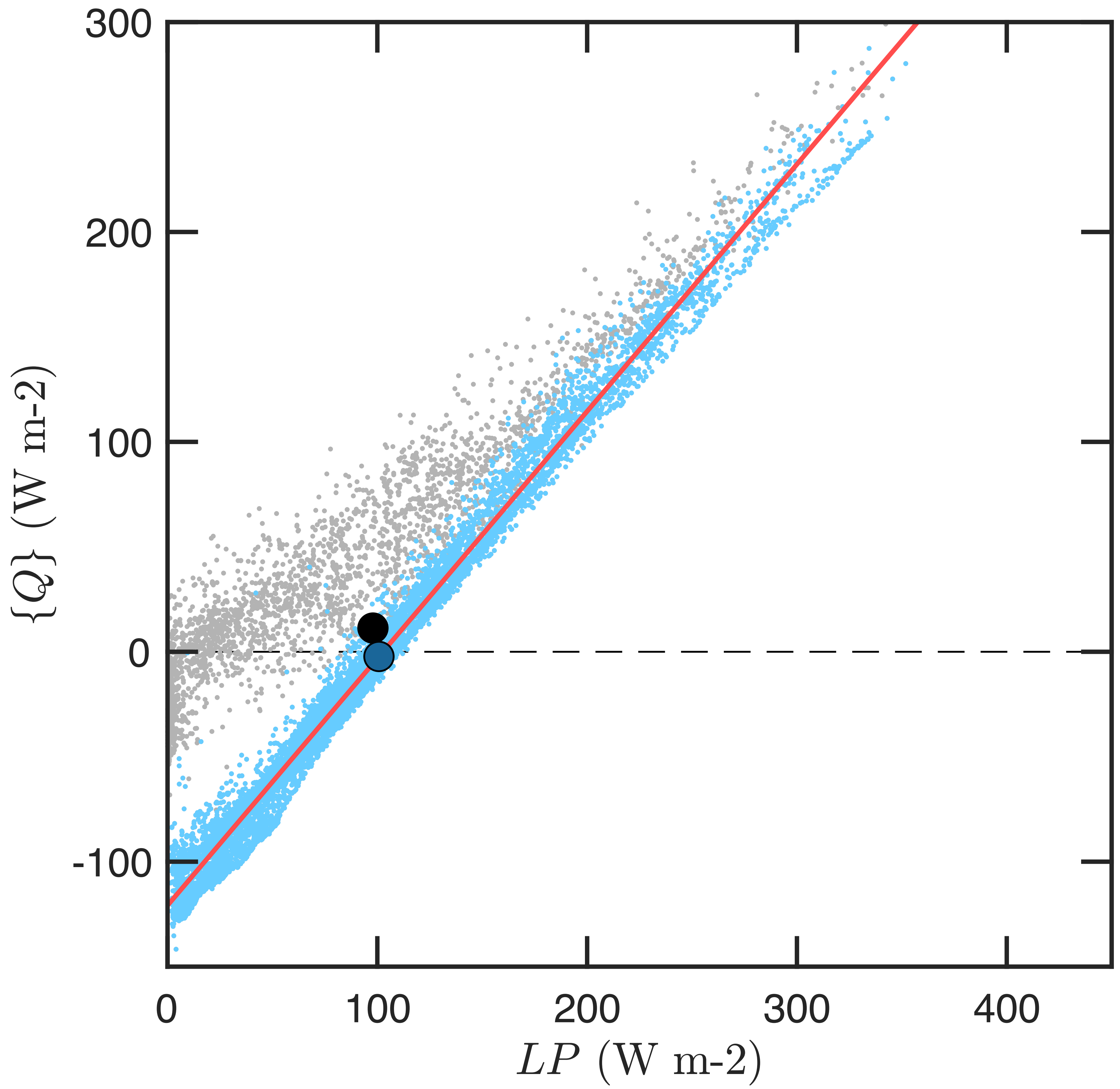


Diabatic heating in the tropics can be decomposed into a cloud component and a residual.

The residual is fairly homogenous and can be interpreted as a clear-sky radiative cooling.

Note that the clear sky cooling is homogenous because of WTG balance.

Radiative Convective Equilibrium



When we look at all points in the tropics, we see a large scatter with regions of heating and cooling.

However, the mean (centroid) of the cloud of points lies at a value of Q1 that is close to zero.

This is especially true for oceanic points.

Radiative Convective Equilibrium

These results imply that the apparent heating averaged over the tropics is nearly zero, i.e:

$$Q_1 \simeq 0$$

What does this mean?

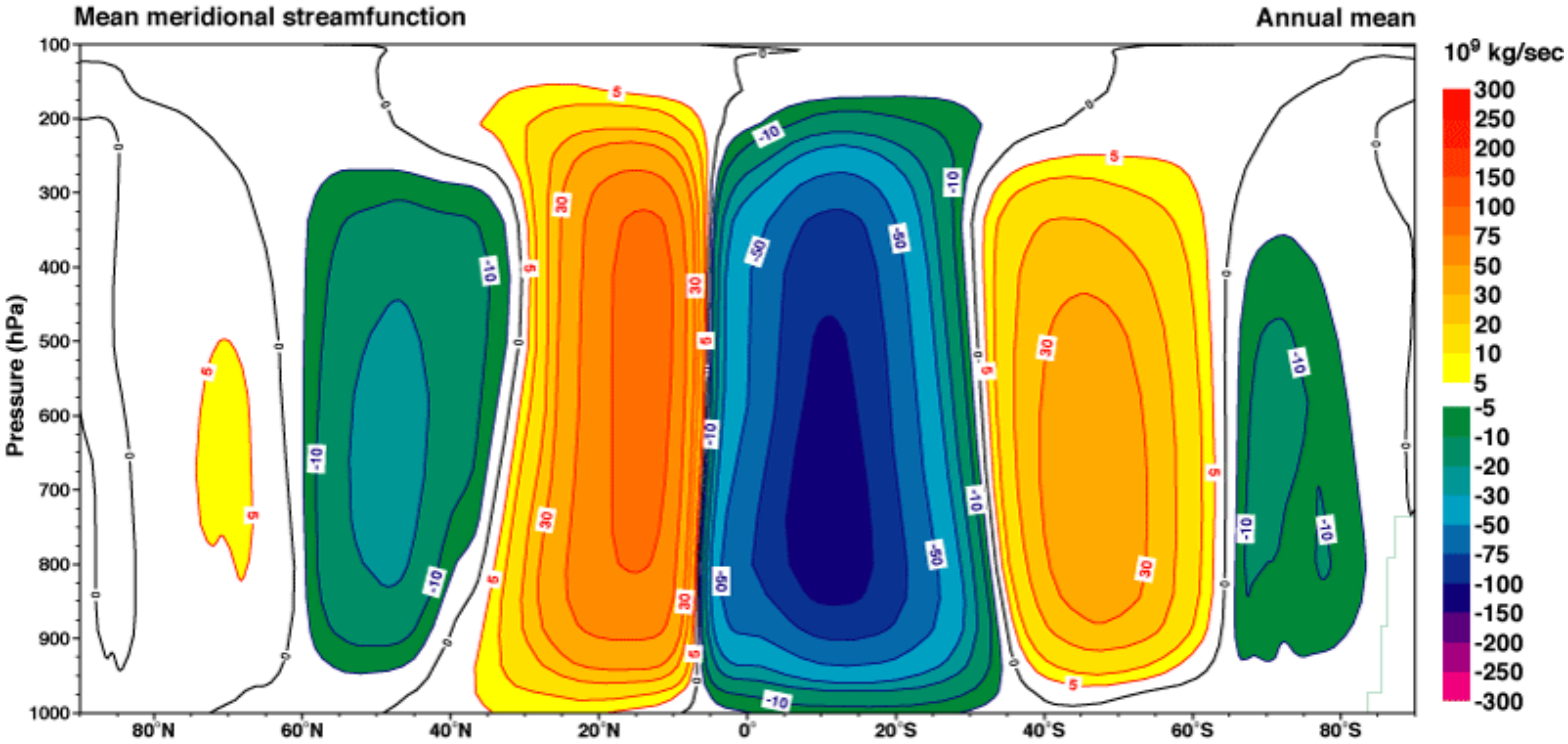
Radiative Convective Equilibrium

If the tropics are in WTG balance, the vertical DSE gradient should be roughly the same everywhere.

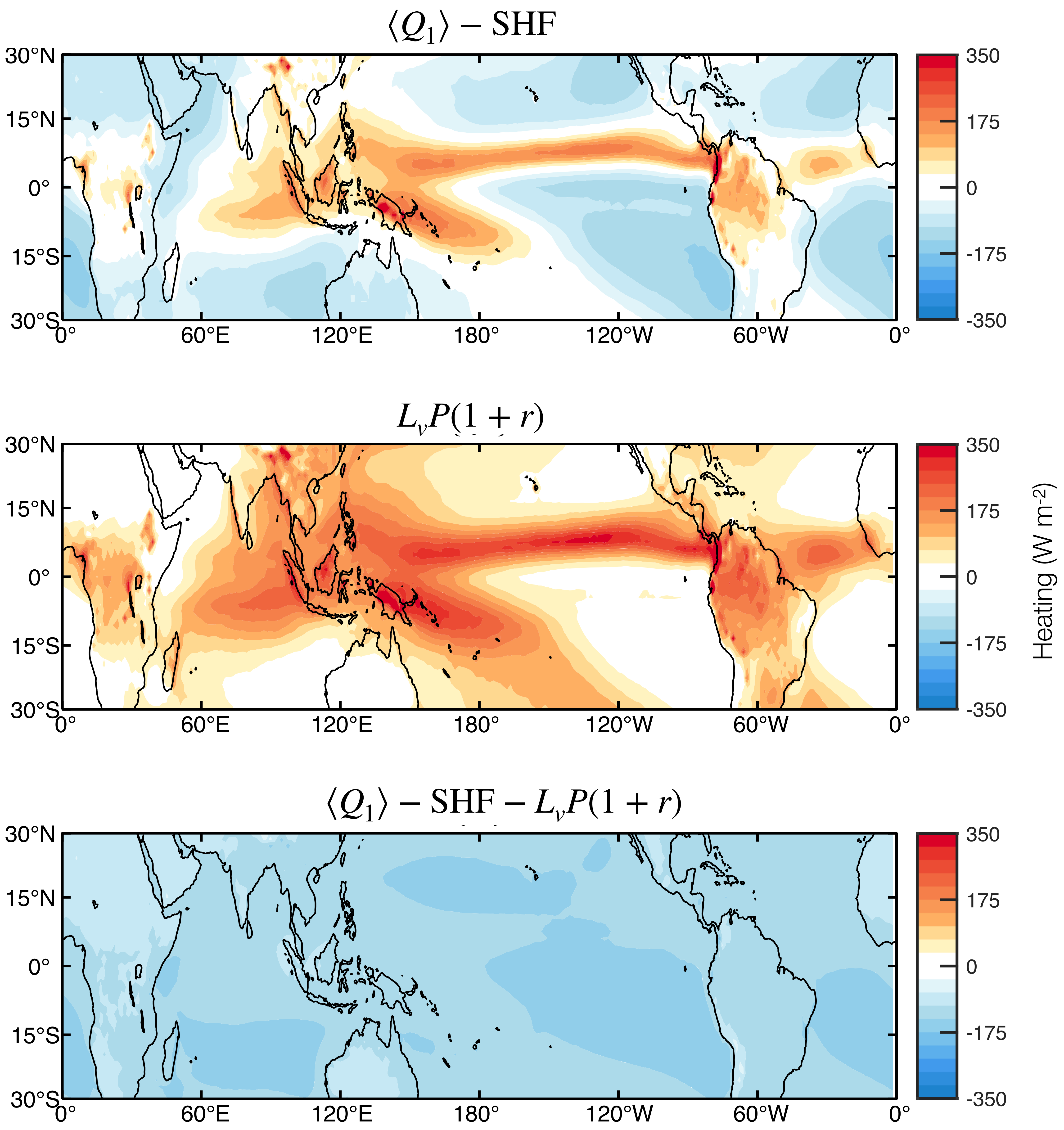
Following mass continuity, the amount of mass that rises must equal the amount of mass that's sinking within the tropics.

$$\omega \frac{\partial \mathrm{DSE}}{\partial p} \simeq Q_1$$

Radiative Convective Equilibrium



Radiative Convective Equilibrium

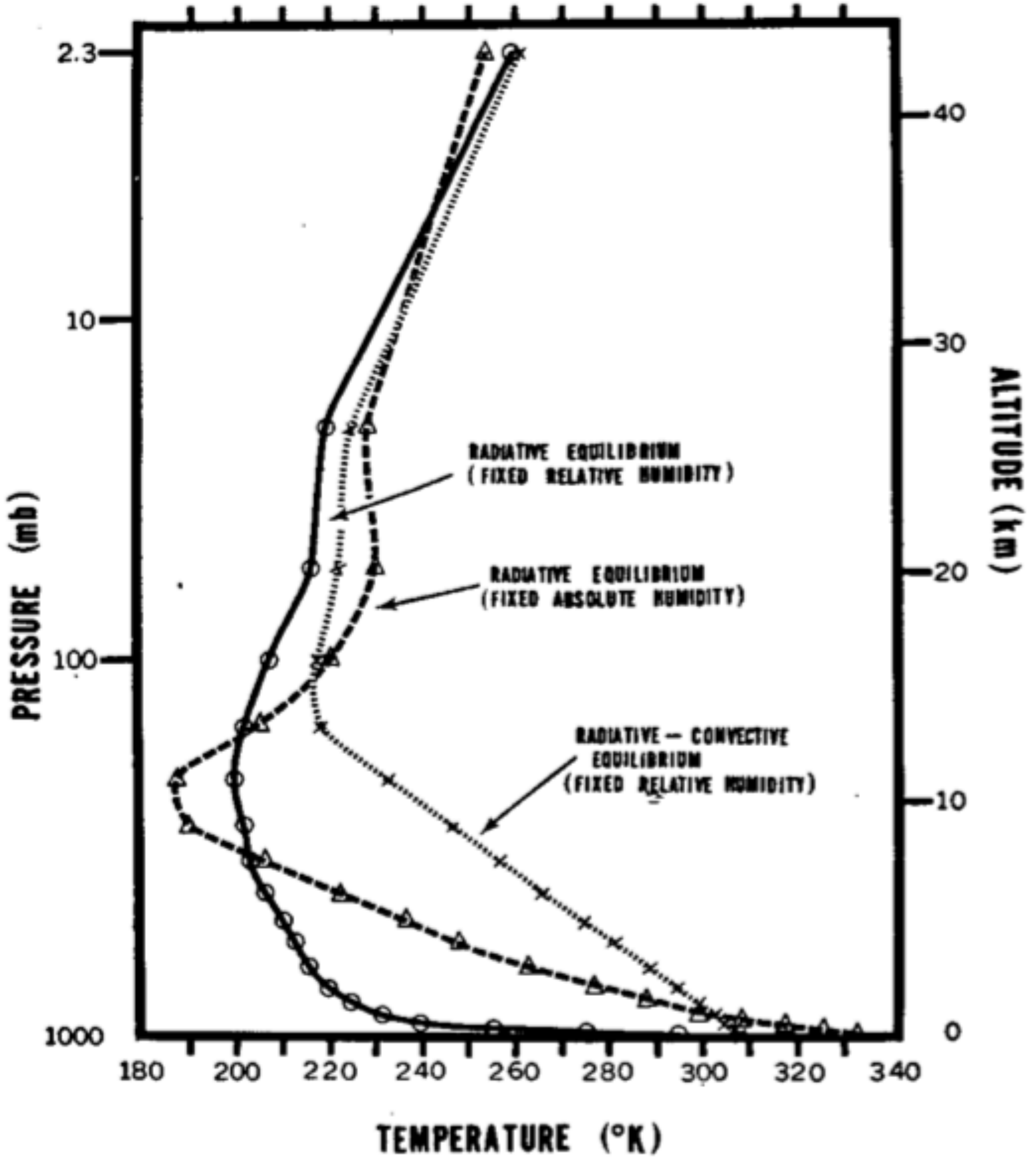


These results imply that the apparent heating averaged over the tropics is nearly zero, i.e:

$$L_v P(1+r) \simeq \langle Q_{r_0} \rangle$$

Cloud heating   Clear sky cooling

Radiative Convective Equilibrium



Radiative-convective equilibrium has been applied to understand the global lapse rate.

But it can also be used to understand the tropics specifically since it behaves like a closed system.