Reynolds Averaging:
Decompose the wind into

$$v = \bar{v} + v'$$

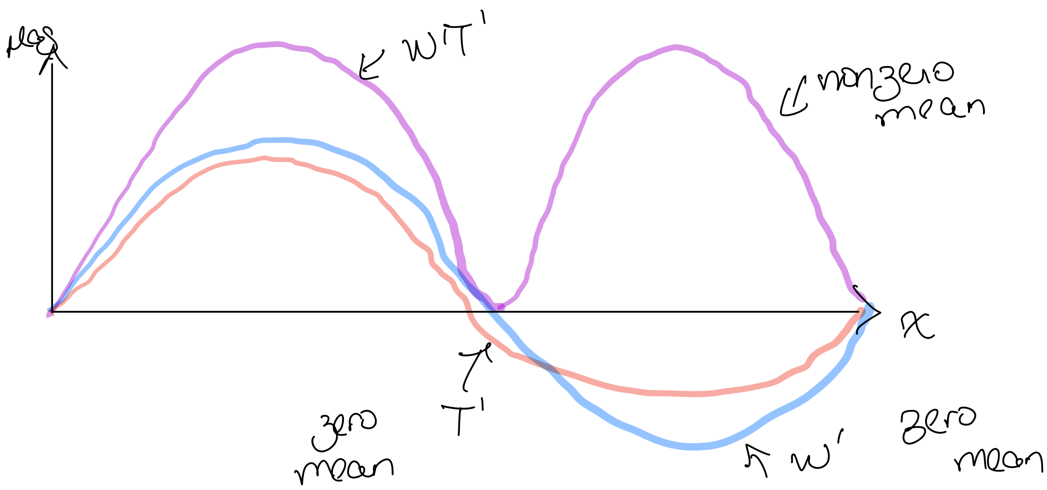
domain average — turbulent flow = fluctuation from mean

$$T = \bar{T} + T' \quad \text{repeat on every variable}$$
$$w = \bar{w} + w'$$

By construction, $\overline{T'} = 0$
" " $\overline{\bar{v}} = \bar{v}$

But, averages of products of turbulent terms may not be zero

$$\overline{w'T'} \neq 0$$



Products of primes $(\overline{w'T'})$ can be understood as covariances.

If we take the thermodynamic eqn.:

$$c_p \frac{\partial T}{\partial t} + c_p \vec{v} \cdot \nabla_h T + \omega \frac{\partial DSE}{\partial p} = Q$$

If we average over a domain, and use the chain rule, assuming Φ doesn't vary much in space, we have:

$$C_p \vec{v} \cdot \nabla_h T + \omega \frac{\partial DSE}{\partial p} \cong \nabla \cdot (\vec{u}\, DSE)$$

$$\vec{u} = u\hat{i} + v\hat{j} + \omega\hat{k}$$

Decomposing and averaging yields the following

$$c_p \frac{\partial \overline{T}}{\partial t} + \vec{v} \nabla_h C_p \overline{T} + \overline{\omega} \frac{\partial \overline{DSE}}{\partial p} = \overline{Q} - \cancel{C_p \nabla_h \cdot \overline{\vec{v}' T'}} - \frac{\partial \overline{\omega' DSE'}}{\partial p}$$

$$\equiv Q_1$$

$$\boxed{Q_1 = \overline{Q} - \frac{\partial \overline{\omega' DSE'}}{\partial p}}$$

Apparent Heating (Yanai et. al. 1973)

We can do the same in the moisture budget:

$$\frac{\partial L_v \overline{q}}{\partial t} + \overline{\vec{v}} \cdot \nabla_h L_v \overline{q} + \overline{\omega} \frac{\partial L_v \overline{q}}{\partial p} = -Q_2$$

$$Q_2 = -L_v \overline{S_q} + \frac{\partial \overline{\omega' L_v q'}}{\partial p}$$

Apparent moisture sink.

From here on we drop the overlines and only use them for the eddy terms.

Radiative-convective equil.

$$\frac{1}{g} \int_0^{p_s} \underbrace{Q_c}_{\text{conv. heating}} \simeq L_v P$$

P = precip.
if clouds don't reside long, the condensation can be assumed to fall as rain.

$$\frac{1}{g}\int_0^{p_s} Q_r = \langle Q_r \rangle$$

column, integrated Q_r