AOS 801: Advanced Tropical Meteorology

Lecture 3   Spring 2023

Moist Thermodynamics

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Photo courtesy of Haochang Luo

Announcements

Homework 1 will be assigned on Monday.

In the meantime, please review all the material we've covered so far. It's a lot!

I will upload several class notes to help you with the review.

Today in the tropics

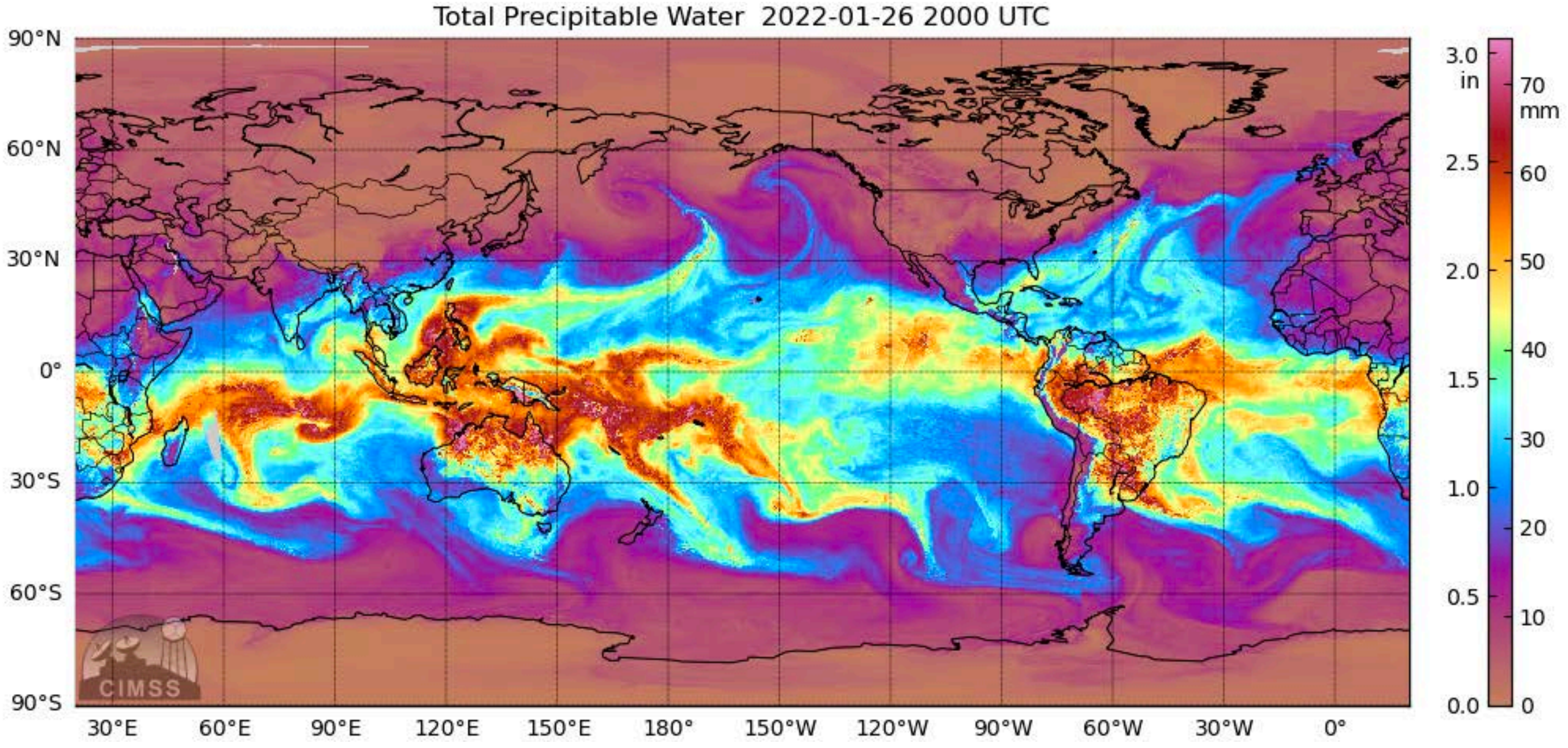
http://tropic.ssec.wisc.edu/real-time/mtpw2/product.php?color_type=tpw_nrl_colors&prod=global2&timespan=24hrs&anim=html5

https://a.atmos.washington.edu/~ovens/wxloop.cgi?ir_moll+/14d/

https://www.cpc.ncep.noaa.gov/products/precip/CWlink/ir_anim_monthly.shtml

https://www.cpc.ncep.noaa.gov/products/precip/CWlink/MJO/obs_phase40_small.gif

$$\langle q \rangle = \frac{1}{g}\int_0^{p_s} q\, dp = \int_0^{\infty} \rho q\, dz$$



Convection Q_c

To understand convection we must invoke the equation for conservation of moisture

$$\frac{Dq}{Dt} = S_q$$

$$S_q = e - c + s - d - \frac{\partial F_q}{\partial p}$$

e = evaporation
c = condensation
s = sublimation
d = deposition

F_q Turbulent flux of moisture

Convection Q_c

$$\frac{Dq}{Dt} = S_q$$

$$S_q = \boxed{e} - c + s - d - \boxed{\frac{\partial F_q}{\partial p}}$$

Includes only evaporation that happens within the parcel

Includes evaporation that occurs as a result of turbulent mixing with a surface of water

Convection Q_c

Need to go back to 1st law to account for condensation

$$C_p \frac{DT}{Dt} = Q + \alpha \frac{Dp}{Dt}$$

$$\frac{Dq}{Dt} = S_q$$

Latent heat is released during condensation

$$Q_c \simeq L_v(c - e)$$

$$S_q \simeq e - c$$

So our heating is negatively related to changes in moisture

$$Q_c \simeq -L_v \frac{Dq}{Dt}$$

The equivalent potential temperature

Need to go back to 1st law to account for condensation

$$C_p dT = -L_v dq_v + \alpha dp$$

$$\frac{Dq_v}{Dt} = S_q$$

Can be written as

$$C_p T d\ln\theta = -L_v dq_v$$

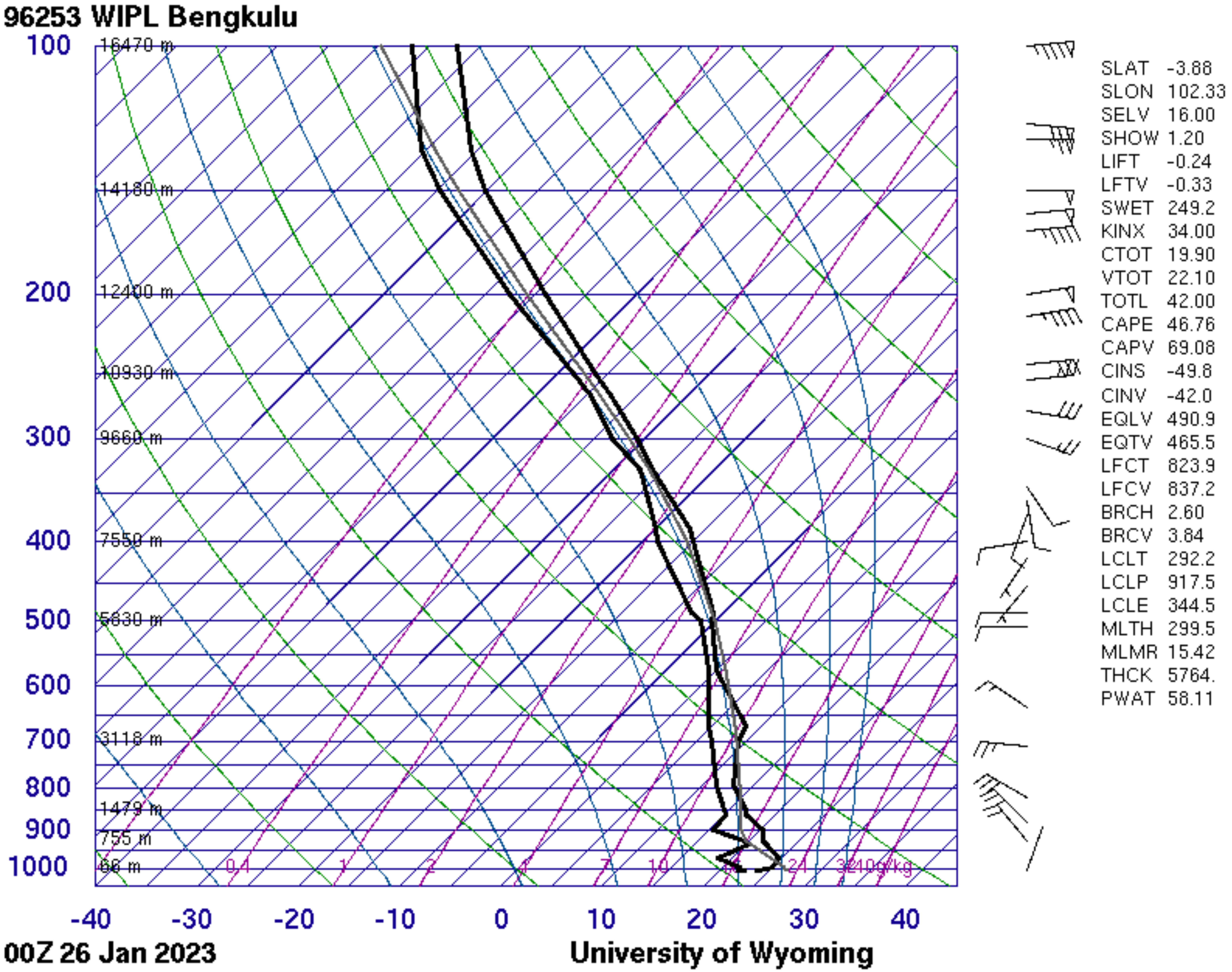
$$S_q = e - c$$

Which can be solved to obtain

$$\theta_e \simeq \theta \exp\left(\frac{L_v q_v}{C_p T}\right)$$

The equivalent potential temperature

The temperature a parcel would have if it condensed all its water vapor and was brought to the surface adiabatically


96253 WIPL Bengkulu
100
200
300
400
500
600
700
800
900
1000
16470 m
14180 m
12400 m
10930 m
9660 m
7550 m
5830 m
3118 m
1479 m
755 m
66 m
-40
-30
-20
-10
0
10
20
30
40
00Z 26 Jan 2023
University of Wyoming
SLAT -3.88
SLON 102.33
SELV 16.00
SHOW 1.20
LIFT -0.24
LFTV -0.33
SWET 249.2
KINX 34.00
CTOT 19.90
VTOT 22.10
TOTL 42.00
CAPE 46.76
CAPV 69.08
CINS -49.8
CINV -42.0
EQLV 490.9
EQTV 465.5
LFCT 823.9
LFCV 837.2
BRCH 2.60
BRCV 3.84
LCLT 292.2
LCLP 917.5
LCLE 344.5
MLTH 299.5
MLMR 15.42
THCK 5764.
PWAT 58.11

Moist entropy

We can define the moist entropy that is using the following form of the first law

$$C_p T d\ln\theta + L_v dq_v = 0$$

Dividing the equation by T yields and rearranging the terms yields the following

$$ds_m = d\left(C_p \ln\theta + \frac{L_v q_v}{T}\right)$$

$$s_m = C_p \ln\theta_e + \text{const}$$

The specific moist entropy

Moist entropy budget

In deriving the equiv. potential temperature and moist entropy we assumed that

$$Q \simeq L_v(c - e) \qquad\qquad dq_v \simeq e - c$$

This is not true. There are multiple sources and sinks of these variables. Hence they are not conserved.

$$\frac{Ds_m}{Dt} = \frac{\dot{Q}_e}{T} \qquad \text{Where} \qquad \dot{Q}_e = -L_f(m - f + s - d) + \dot{Q}_r + -\frac{\partial}{\partial p}(F_q + F_T)$$

e = evaporation
c = condensation
s = sublimation
d = deposition
m = melting
f = freezing

The moist static energy budget

$$C_p \frac{DT}{Dt} - \alpha \frac{Dp}{Dt} = -L_v \frac{Dq_v}{Dt} + \dot{Q}_e$$

We can get a simpler conserved variable is we assume hydrostatic balance:

$$\frac{Dp}{Dt} \simeq \frac{dp}{dz}\frac{Dz}{Dt} = -\rho g \frac{Dz}{Dt} = -\rho \frac{D\Phi}{Dt}$$

In HW1 you will examine the validity of this approximation.

Which we can plug into the first equation above to obtain

$$\frac{D\mathrm{MSE}}{Dt} = \dot{Q}_e$$

Where

$$\mathrm{MSE} = c_p T + \Phi + L_v q_v$$

The moist static energy

Relationship between MSE and moist entropy

Given that $$\frac{Ds_m}{Dt} = \frac{\dot{Q}_e}{T}$$ and $$\frac{D\text{MSE}}{Dt} = \dot{Q}_e$$

It follows that $$\frac{Ds_m}{Dt} = \frac{1}{T}\frac{D\text{MSE}}{Dt}$$ $$Tds_m = d\text{MSE}$$

Also recall that $$ds_m = C_p d \ln \theta_e$$

The **MSE**, **moist entropy** and θ_e are interrelated

Maxwell's Relation

We can obtain several relations using MSE or moist entropy that will prove useful in this class

$$T ds_m = d\mathrm{MSE} = C_p dT + d\Phi + L_v dq$$

If we keep pressure constant:

$$T(ds_m)_p = C_p dT + L_v dq = (d\varepsilon_m)_p$$

If moist entropy is kept constant

$$(d\varepsilon_m)_{s_m} = \alpha (dp)_{s_m}$$

Cross differentiation yields

$$\left(\frac{dT}{dp}\right)_{s_m} = \left(\frac{d\alpha}{ds}\right)_p$$

The moist adiabatic lapse rate

A parcel that rises moist adiabatically conserves its MSE as it rises

$$\frac{D\mathrm{MSE}}{Dz} \approx 0$$

By expanding the definition and after some algebra and rearranging, we can obtain the moist adiabatic lapse rate

$$\frac{dT}{dz} = -\Gamma_m \qquad \Gamma_m = \Gamma_d \frac{1 + \frac{L_v q_s}{R_d T}}{1 + \frac{L_v^2 q_s}{c_p R_v T^2}} \qquad \Gamma_d = \frac{g}{C_p}$$

Is the **moist adiabatic lapse rate.**