Moist thermodynamics:

$$C_p T \, d\ln\theta + L_v \, dq = 0$$

$$d\ln\theta + \frac{L_v}{C_p T} dq = 0$$

* Changes in $T \ll$ changes in q
* q changes following Clausius-Clapeyron

$$\therefore \frac{L_v}{C_p T} dq \simeq d\left(\frac{L_v q}{C_p T}\right)$$

So that the thermodynamic budget becomes:

$$d\underbrace{\left(\ln\theta + \frac{L_v}{C_p T} q\right)}_{} = 0$$

Can be integrated to get

$$\theta_e = \theta \exp\left(\frac{L_v q}{C_p T}\right)$$

But you can also define

$$S_m = C_p \ln\theta + \frac{L_v q}{T}$$ **Moist Entropy**

$$S_m = C_p \ln\theta_e$$

There's a third variable that describes moist thermo:

let's go back to the first law:

$$C_p \frac{DT}{Dt} - \omega\alpha = -L_v \frac{Dq}{Dt} \qquad \frac{\partial\Phi}{\partial p} = -\alpha$$

$$\underline{C_p \frac{DT}{Dt} + \omega \frac{\partial\Phi}{\partial p}} = -L_v \frac{Dq}{Dt} \qquad \Phi = gz$$

$$\alpha = 1/\rho$$

From last class: $\frac{DDSE}{Dt} = -L_v \frac{Dq}{Dt}$ $\quad DSE = C_pT + \Phi$

Define MSE : moist static energy

$\frac{DMSE}{Dt} \simeq 0$ for moist, hydrostatic adiabatic processes

$MSE = DSE + L_v q$

Maxwell's Relations:

$$T\,dS_m = dMSE = C_p dT + d\Phi + L_v dq - \alpha dp$$

What if changes occur while S_m is constant:

$$0 = C_p dT + d\Phi + L_v dq$$

Moist Enthalpy $\quad \mathcal{E}_m = C_pT + L_v q$

$$d\mathcal{E}_m = -d\Phi = \alpha dp$$

You can get that $\left(\frac{d\mathcal{E}_m}{dp}\right)_{S_m} = \alpha$

S_m is held constant

What if we keep pressure constant: $(\alpha dp = 0)$

$$T\,dS_m = C_p dT + L_v dq$$

$$T\,dS_m = d\mathcal{E}_m$$

$$\left(\frac{d\mathcal{E}_m}{dS_m}\right)_p = T$$

Let's cross differentiate

$$\left(\frac{d}{dp}\right)_{S_m}\left(\frac{d\mathcal{E}_m}{dS_m}\right)_p = \left(\frac{dT}{dp}\right)_{S_m} \longrightarrow \left(\frac{d}{dS_m}\right)_p\left(\frac{d\mathcal{E}_m}{dp}\right)_{S_m} = \left(\frac{d\alpha}{dS_m}\right)_p$$

are the same

$$\left(\frac{dT}{dp}\right)_{S_m} = \left(\frac{d\alpha}{dS_m}\right)_p$$ Maxwell's Relation