AOS 801: Advanced Tropical Meteorology

Lecture 2 Spring 2023

Review of Dynamics and Thermodynamics

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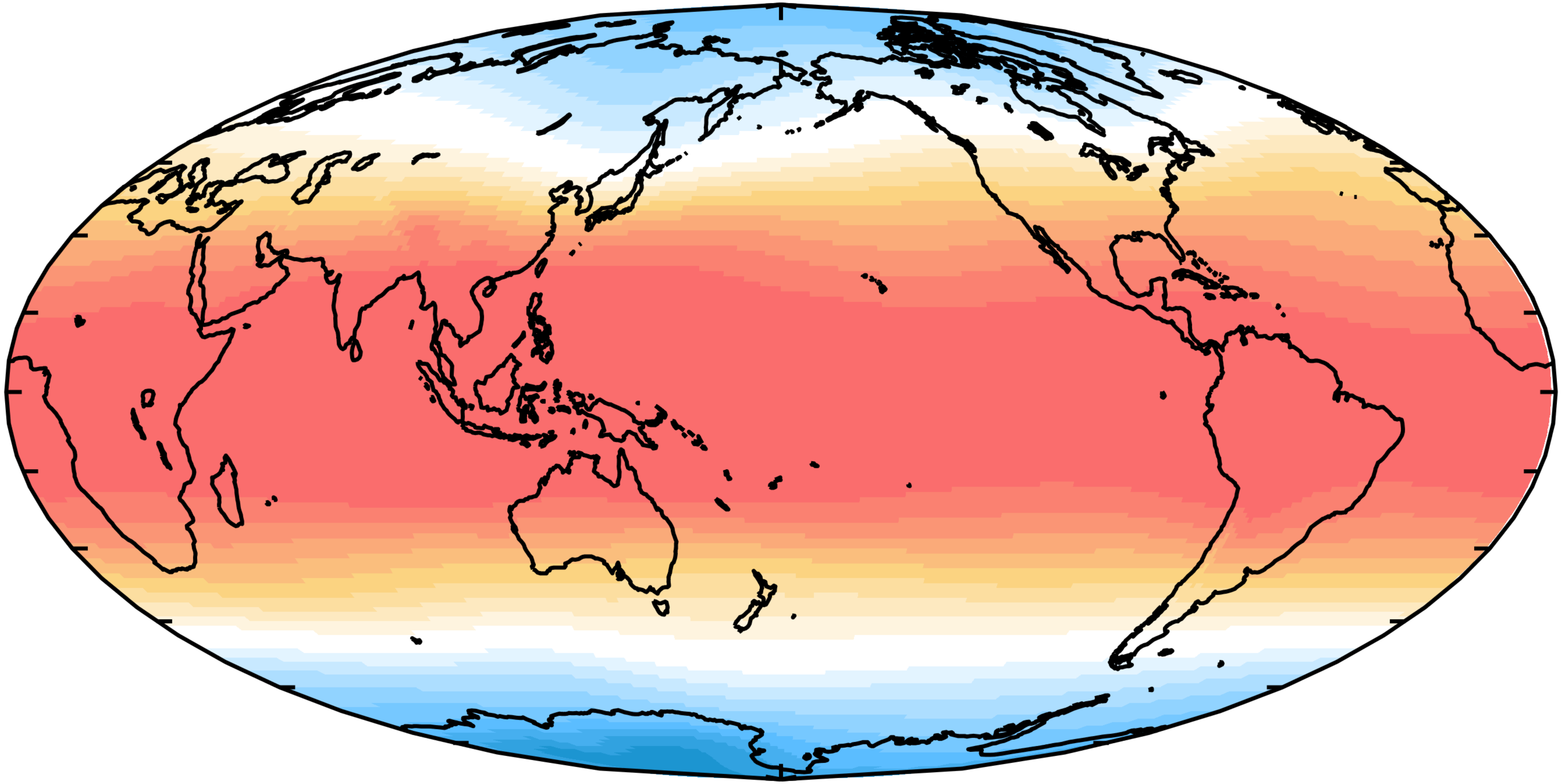
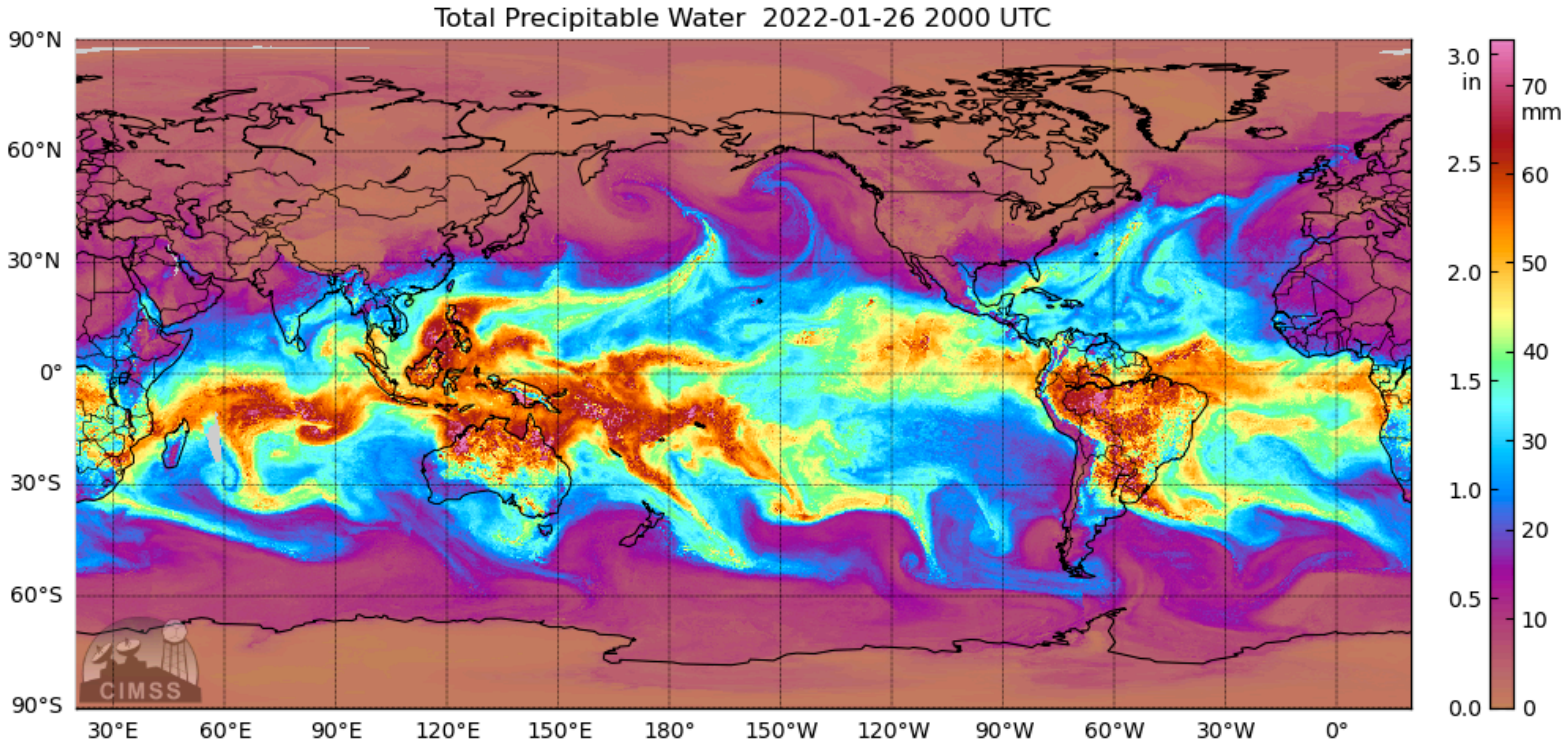
Homogeneous temperatures

T_{500}



Lots of water vapor



Momentum Equations in Spherical Coordinates

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho a\cos\phi}\frac{\partial p}{\partial\lambda} + 2\Omega v\sin\phi + 2\Omega w\cos\phi + F_{rx}$$

$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + \frac{uv}{a} = -\frac{1}{\rho a}\frac{\partial p}{\partial\phi} - 2\Omega u\sin\phi + F_{ry}$$

$$\frac{Dw}{Dt} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\phi + F_{rz}$$

Momentum Equations in Spherical Coordinates

A qualitative scale analysis will quickly reveal that the metric terms, the "non-traditional" Coriolis terms, and the molecular friction are negligibly small.

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho a\cos\phi}\frac{\partial p}{\partial \lambda} + 2\Omega v\sin\phi + 2\Omega w\cos\phi + F_{rx}$$

$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho a}\frac{\partial p}{\partial \phi} - 2\Omega u\sin\phi + F_{ry}$$

$$\frac{Dw}{Dt} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\phi + F_{rz}$$

Momentum Equations: Tangent Plane Approximation

The resulting system of equations are equivalent to analyzing them on a "tangent plane".

Scalar form

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Vector form

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k} - f\mathbf{k}\times\mathbf{v}$$

$$\mathbf{u} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

$$\mathbf{v} = u\mathbf{i} + v\mathbf{j}$$

Primitive equations for an ideal gas

Momentum
$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k} - f\mathbf{k} \times \mathbf{v}$$

Mass Continuity
$$\frac{D\rho}{Dt} = -\rho\nabla \cdot \mathbf{u}$$

Thermodynamic
$$C_p\frac{DT}{Dt} - \alpha\frac{Dp}{Dt} = Q$$

Gas State
$$p = \rho R_a T \qquad p\alpha = R_a T$$

Primitive Equations: Pressure Coordinates

Momentum $$\frac{D\mathbf{v}}{Dt} = -\nabla_h \Phi - f\mathbf{k} \times \mathbf{v}$$ $$\frac{\partial \Phi}{\partial p} = -\alpha$$

Mass Continuity $$\frac{\partial \omega}{\partial p} = -\nabla_h \cdot \mathbf{v}$$

Thermodynamic $$C_p \frac{DT}{Dt} - \omega\alpha = Q$$

Gas State $$p\alpha = R_a T$$

In atmospheric sciences, it is very common to use pressure as a vertical coordinate.

It gets rid of density!

$$\omega = \frac{Dp}{Dt} \qquad \Phi = gz$$

Thermodynamic equation in a hydrostatic atmosphere

A particularly useful form of the thermodynamic budget can be obtained if we apply hydrostatic balance:

$$C_p \frac{DT}{Dt} - \omega\alpha = Q \qquad \frac{\partial \Phi}{\partial p} = -\alpha$$

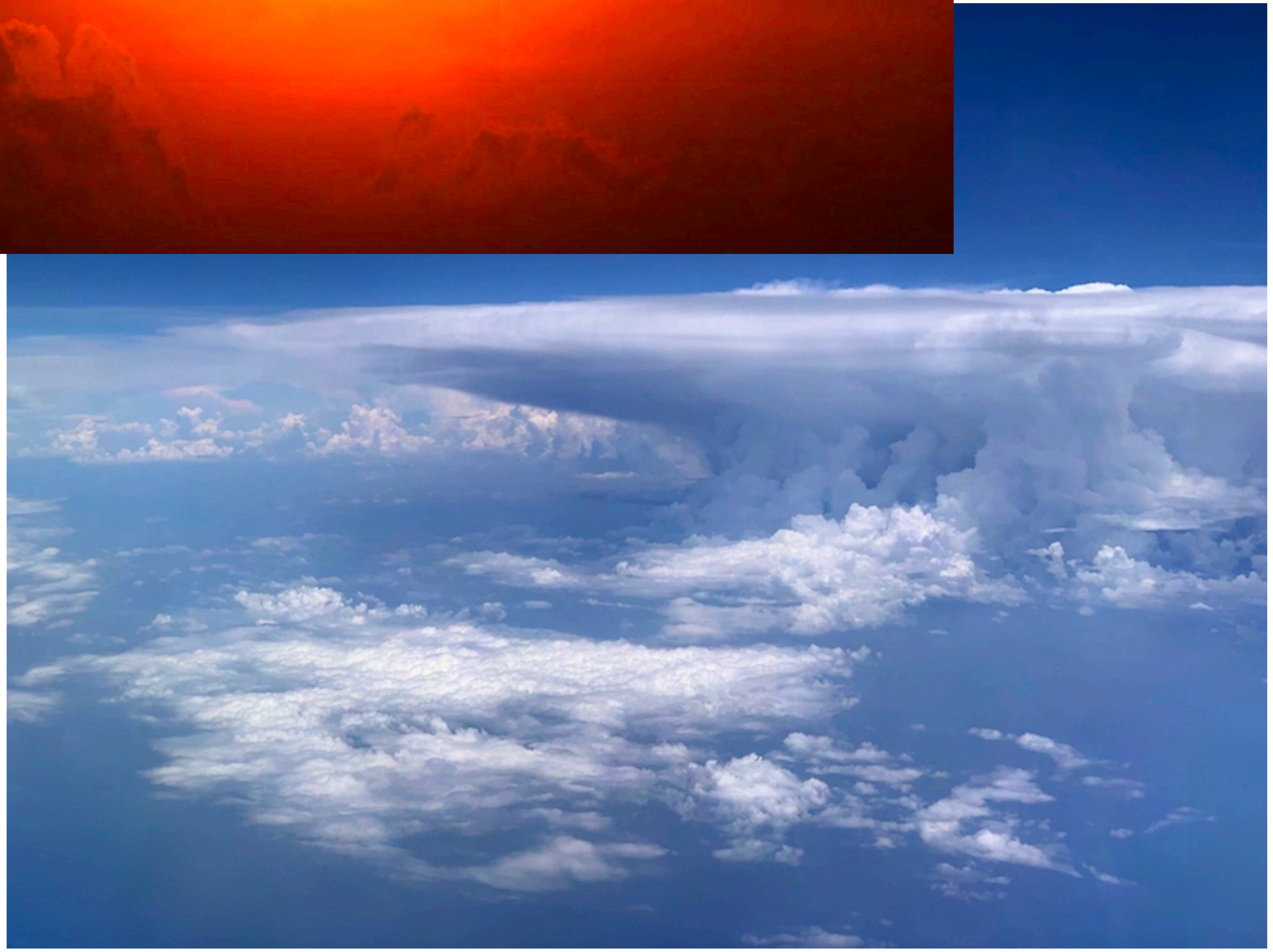
Using hydrostatic balance to remove the specific volume yields:

$$C_p \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla_h C_p T + \omega \frac{\partial \mathrm{DSE}}{\partial p} = Q\,.$$

Where $\mathrm{DSE} = C_p T + \Phi$ is the **dry static energy**, a measure of entropy in a hydrostatic atmosphere.

But wait!!

$$C_p \frac{DT}{Dt} - \omega\alpha = \boxed{Q}$$



In a dry atmosphere the primitive equations are enough to describe the evolution of systems.

But we have diabatic heating here…

But wait!!

$$Q = Q_c + Q_r$$

In most of your dynamics classes we ignore convection and radiation. In the tropics these are the primary drivers of the circulation.

What do we need to understand convection?

Water vapor

Water vapor is roughly an ideal gas. It follows Dalton's law of partial pressures (the total pressure is the sum of the pressure of all the constituent gases).

$$e\alpha_v = R_v T$$

The mixing ratio is the amount of water vapor mass per unit of dry air

$$r_v = \frac{m_v}{m_d}$$

The specific humidity is the amount of water vapor per unit of total air mass.

$$q_v = \frac{m_v}{m_d + m_v} = \varepsilon \frac{e}{p_d + e}$$

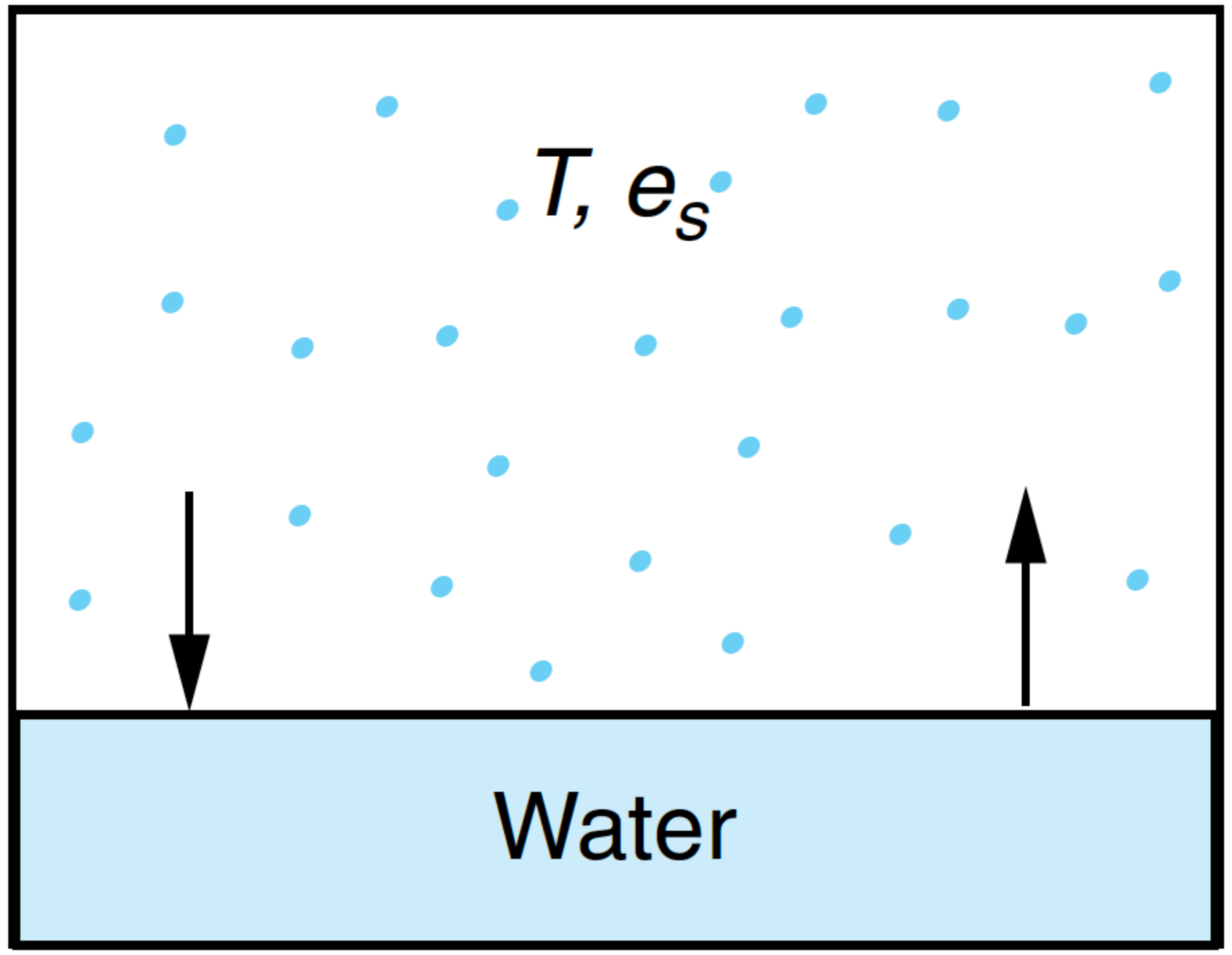
$$q_v \simeq r_v$$

Saturation

The vapor pressure for equilibrium is known as the saturation vapor pressure e_s

$$\mathrm{RH} = \frac{e}{e^*}$$

If the RH is 100%, you have reached equilibrium. That is, the rate of evaporation to condensation is the same+



(b) Saturated

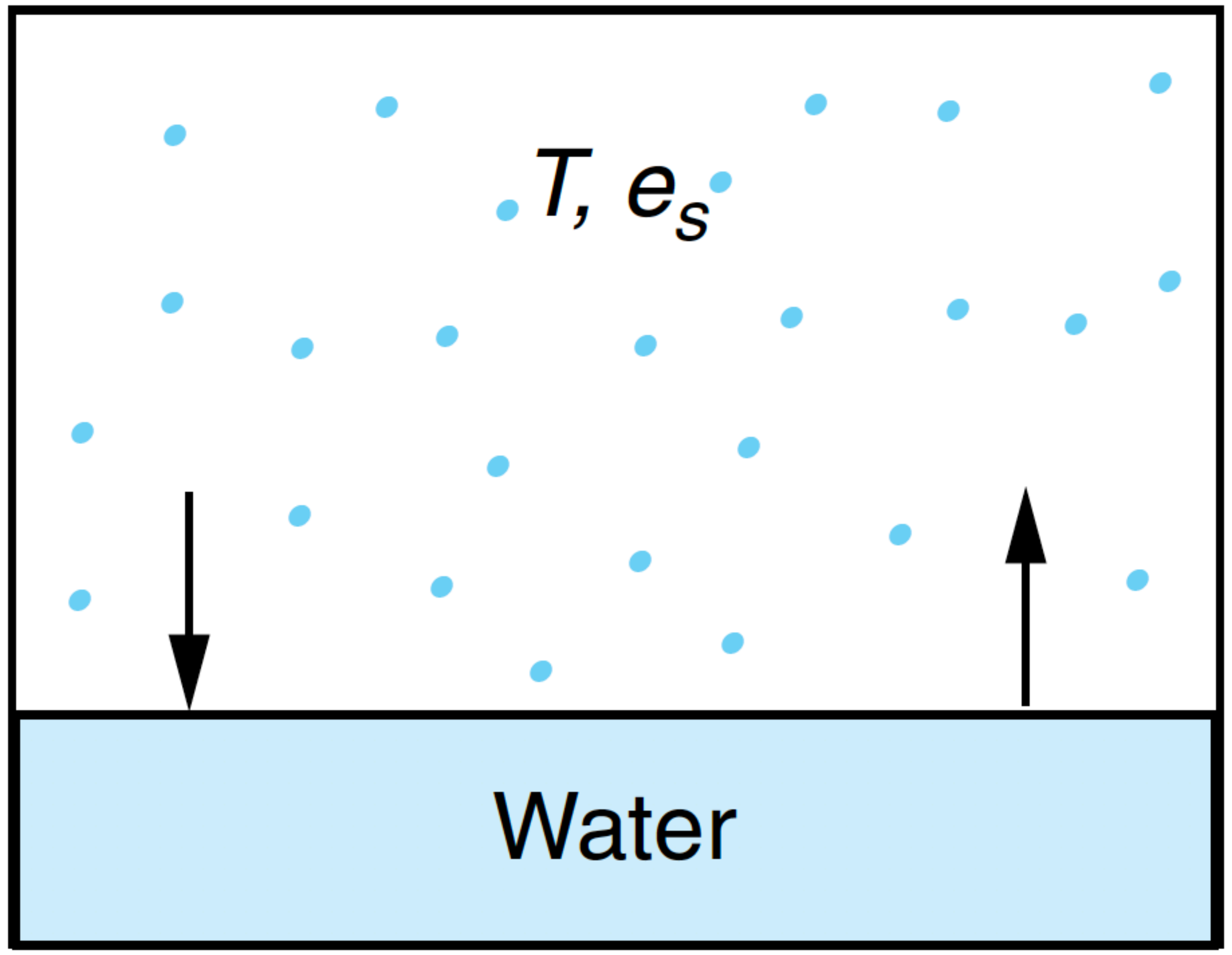
Wallace and Hobbs (2006)

(+This only applies for flat surfaces of water. The story is different for cloud droplets)

The Clausius-Clapeyron equation

Saturation vapor pressure is a function of temperature only in the atmosphere.

$$\frac{1}{e^*}\frac{de^*}{dT} \simeq \frac{L_v}{R_v T^2}$$



Wallace and Hobbs (2006)

Yields a solution of the form

$$e^* \simeq e_0^* \exp\left(\frac{L_v}{R_v}\left[\frac{1}{T_0} - \frac{1}{T}\right]\right)$$

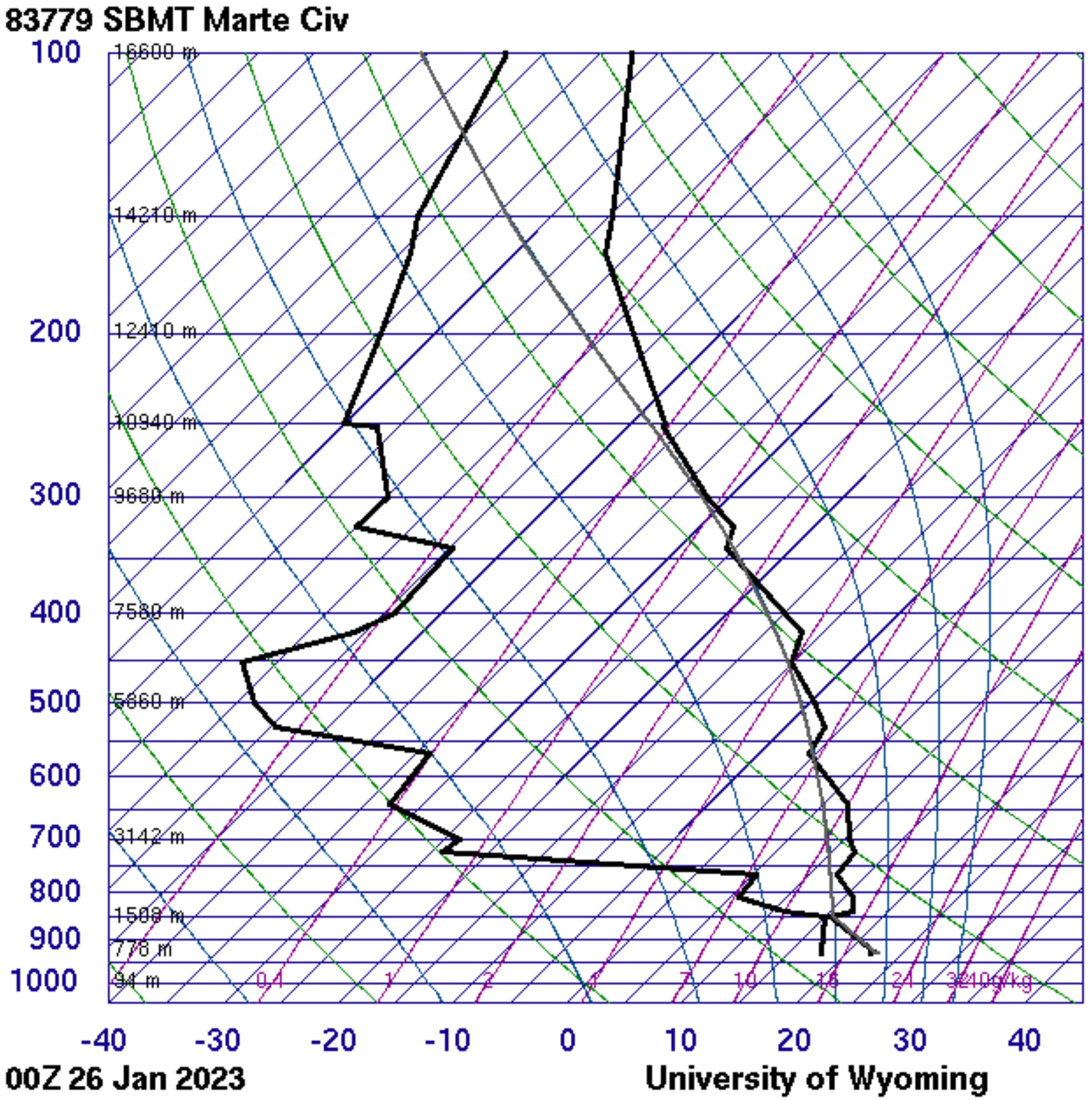
A simplified version of the solution takes the following form

$$e^* = A \exp\left(-\frac{B}{T}\right)$$

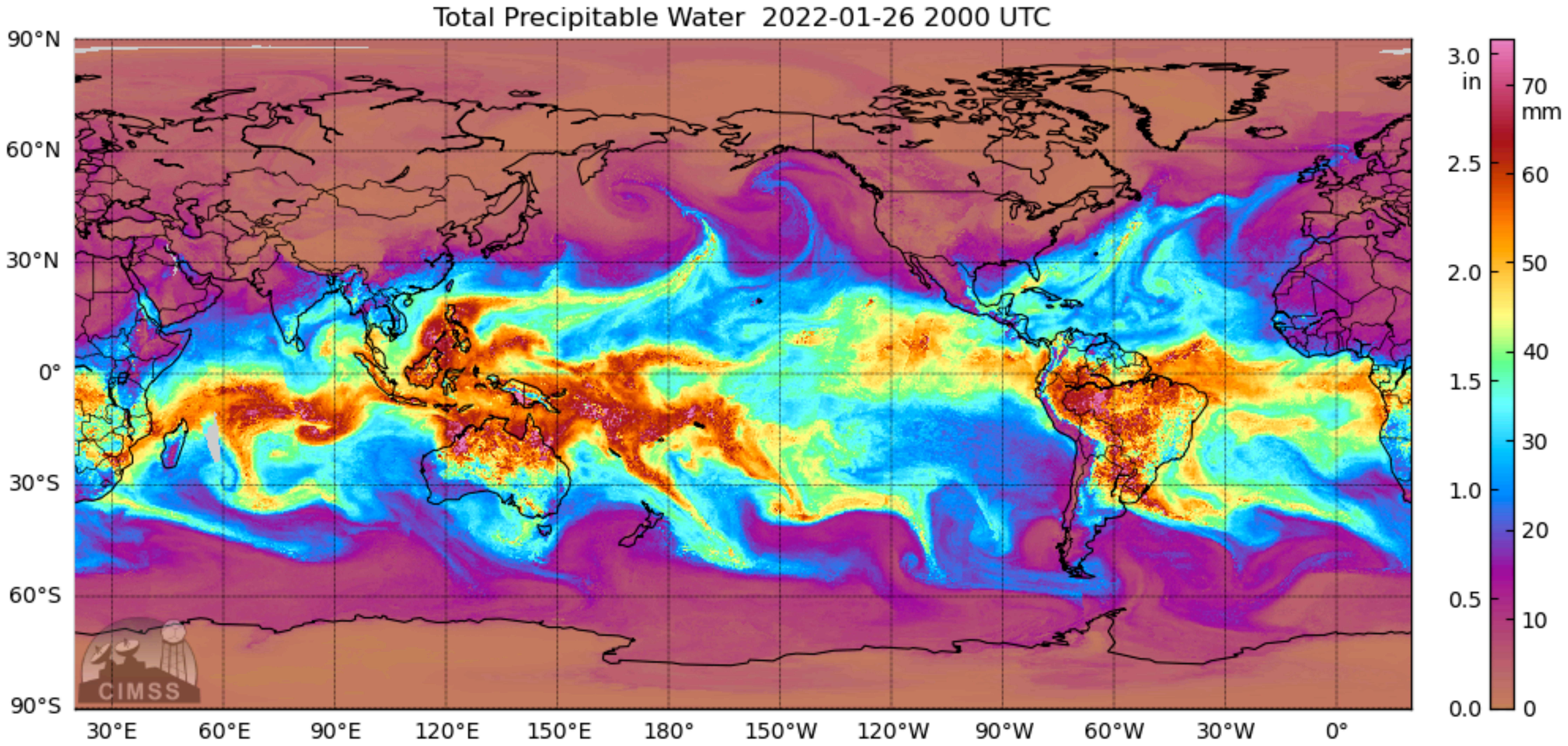
$$A = 2.53 \times 10^9 hPa$$

$$B = L_v/R_v = 5.42 \times 10^3 K$$

But the relative humidity changes in space and time in the tropics



The Clausius-Clapeyron equation



Convection Q_c

To understand convection we must invoke the equation for conservation of moisture

$$\frac{Dq}{Dt} = S_q$$

$$S_q = e - c + s - d - \frac{\partial F_q}{\partial p}$$

e = evaporation
c = condensation
s = sublimation
d = deposition

F_q Turbulent flux of moisture

Convection Q_c

$$\frac{Dq}{Dt} = S_q$$

$$S_q = \boxed{e} - c + s - d - \boxed{\frac{\partial F_q}{\partial p}}$$

Includes only evaporation that happens within the parcel

Includes evaporation that occurs as a result of turbulent mixing with a surface of water